0.2) *A short revision about second degree Polynomials*

Theory

A second degree polynomial is : $p(x) = ax^2 + bx + c$ (a, b, c *coefficients* $\in \mathbb{R}, a \neq 0$)
Let us consider the equation $p(x) = 0$.

The solutions are $\boxed{\begin{array}{l} x_1 \\ x_2 \end{array} = \frac{-b \mp \sqrt{\Delta}}{2a}}$ are real for $\Delta \geqslant 0$ (Proof here)

De Viète formulae for the sum (s) and the product(p):

$$\boxed{s = \frac{-b}{a} \qquad p = \frac{c}{a}}$$

Exercises

1) Prove De Viète formulae

2) Solve the equations i) $x^2 - 10x + 21 = 0$
ii) $x^4 - 10x^2 + 21 = 0$
iii) $\log(x)\left[\log(x) + \frac{1}{\log(x)}\right] + \log(\frac{1}{x^{10}}) = -20$ where log is $\log_{10}$

3) i) Using a calculator, find $\log_{4-\sqrt{15}}(31 - 8\sqrt{15})$
ii) Show that the equation E: $(4+\sqrt{15})^x + (4-\sqrt{15})^x = 62$
is equivalent to an equation $a\xi^2 + b\xi + c$ with $\xi = (4-\sqrt{15})^x$

iii) Hence solve E.

Answers

1) s = $\frac{-b+\sqrt{\Delta}}{2a} + \frac{-b-\sqrt{\Delta}}{2a}$ and p = $\frac{-b+\sqrt{\Delta}}{2a} \cdot \frac{-b-\sqrt{\Delta}}{2a}$

2) i) S=$\{3, 7\}$ ii) S=$\left\{\pm\sqrt{3}, \pm\sqrt{7}\right\}$ iii) $S = \{10^3, 10^7\}$

3) i) $\log_{4-\sqrt{15}}(31 - 8\sqrt{15}) = \frac{\ln(31-8\sqrt{15})}{\ln(4-\sqrt{15})} = 2$
ii) it is equavalent to $\xi^2 - 62\xi + 1 = 0$ where $\sqrt{\Delta} = 16\sqrt{15}$
iii) then $(4-\sqrt{15})^x = 31 \pm 8\sqrt{15}$
considering the solution with sign $'-'$ (can you tell why ?) :
$x = \log_{4-\sqrt{15}}(31 - 8\sqrt{15}) = 2$.