IB$_2$ **Test 2** Thursday 31 Oct 2025

Subject : *Antiderivatives & Integrals* ANSWERS

Tot: /32

Problem 1 [12 marks]

1) $\int 3\cos(x) - 4\cos(4x)dx = \boxed{3\sin(x) + \cos(4x) + c}$

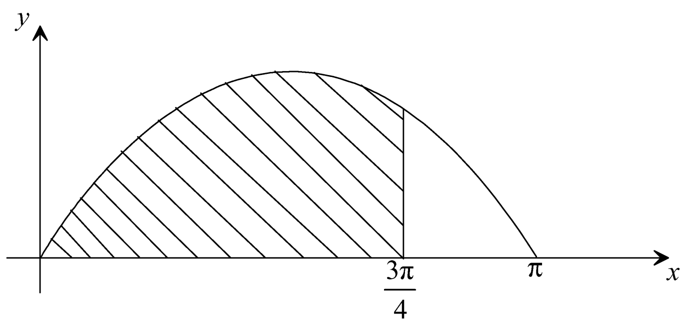
3) $\int 3e^{\ln(x)} \cdot \frac{1}{x} dx = \boxed{3e^{\ln(x)} + c}$

2) $\int \cos(x^3+1)3x^2dx = \boxed{\sin(x^3+1)+c}$

4) $\int \left(\frac{16}{x^5} - \frac{4}{x^3} - \frac{2}{x}\right)dx = \boxed{\frac{-4}{x^4} + \frac{2}{x^2} - 2\ln(x) + c}$

Problem 2 (IB Paper1 question) [6 marks]

The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \frac{3\pi}{4}$.



Given that $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

$$A = \int_0^{\frac{3\pi}{4}} \sin(x)dx = \Big[-\cos(x)\Big]_0^{\frac{3\pi}{4}} = -\cos(\tfrac{3\pi}{4}) + \cos(0) = \boxed{\frac{\sqrt{2}}{2} + 1}$$

Problem 3 [14 marks]

Give the **exact** expression for the area of the 3 surfaces below

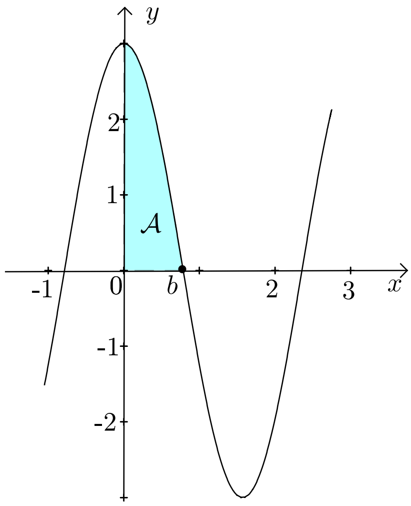
1) $f(x) = 3\cos(2x)$

$a = 0$
$b > 0$: first intersection with the x − axis

b is the first positive solution of $f(x) = 0$
$\cos(2x) = 0 \Rightarrow b = \frac{\pi}{4}$

$$\mathcal{A} = 3\int_0^{\frac{\pi}{4}} \cos(2x)dx = \frac{3}{2}\Big[\sin(2x)\Big]_0^{\frac{\pi}{4}} = \frac{3}{2}(1-0)$$

$$= \boxed{\frac{3}{2}}$$

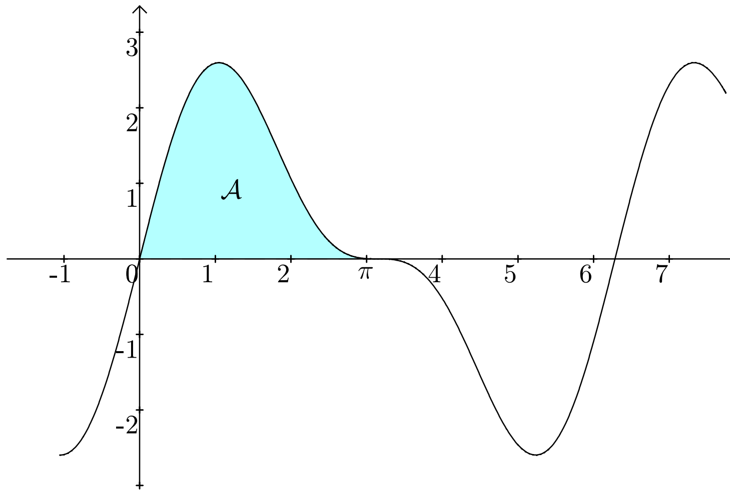


2) $f(x) = \sin(2x) + 2\sin(x)$

$a = 0$
$b = \pi$

$$\mathcal{A} = \int_0^{\pi} (\sin 2x + 2\sin x)dx$$
$$= \Big[-\frac{\cos(2x)}{2} - 2\cos(x)\Big]_0^{\pi}$$
$$= \left(-\frac{1}{2} + 2\right) - \left(-\frac{1}{2} - 2\right)$$

$$= \boxed{4}$$



3)

i) The *derivative* of $F(x) = x(\ln(x) - 1)$
is $f(x) = F'(x) = \ln(x)$

ii) Hence (by definition of antiderivative)
$F(x) + c$ is an *antiderivative* of $f(x) = \ln(x)$

iii) The area under the curve $y = \ln(x)$
for $a < x < b$ with $a = 1$ and $b = e$
is

$$\mathcal{A} = \int_1^e f(x)dx = \Big[F(x)\Big]_0^e$$
$$= \ln(e) - \ln(0)$$
$$= \boxed{1}$$