Test 1

November 20. 2024

Maths IB_2

`subjets` : *Maximum and Minimum* & Optimisation,

Tot :[/ 28 marks]

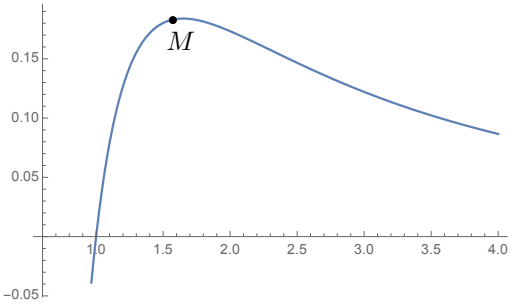
ANSWERS

Problem 1 *[/ 6 marks]*

Let $g(x) = \frac{\ln(x)}{x^2}$

1. $g'(x) = \frac{\frac{1}{x}x^2 - \ln(x)2x}{(x^2)^2} = \frac{1 - 2\ln(x)}{x^3}$

2. The figure below shows the graph of g.

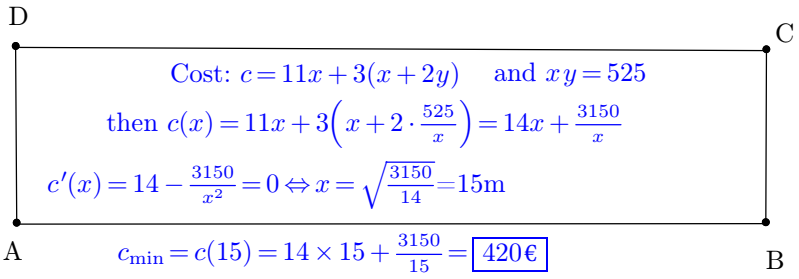


Find the exact coordinates of the maximum M.

$g'(x) = 0 \Leftrightarrow 1 - 2\ln(x) = 0 \Leftrightarrow x = e^{\frac{1}{2}}$ and $y = g(e^{\frac{1}{2}}) = \frac{\ln\left(e^{\frac{1}{2}}\right)}{\left(e^{\frac{1}{2}}\right)^2} = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$ $\boxed{\text{M}:(\sqrt{e}, \frac{1}{2e})}$

Problem 2 *[/ 9 marks]*

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.



The fencing used for side AB costs 11€ per metre. The fencing for the other three sides costs 3€ per metre. The farmer creates an enclosure so that the cost is a minimum.
Find this minimum cost.

Problem 2 *[/ 9 marks]*

The position of a particle is given by $x(t) = \frac{e^t}{t}$

1) Using your calculator, give the *displacement* of the particle between $t_1 = 1s$ and $t_2 = 4s$

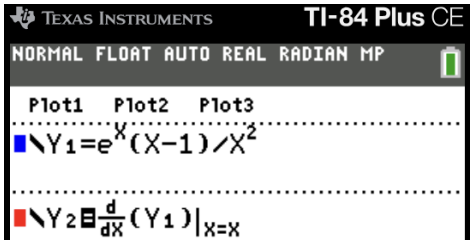
2) Find and expresssion for the *velocity* of the particle.

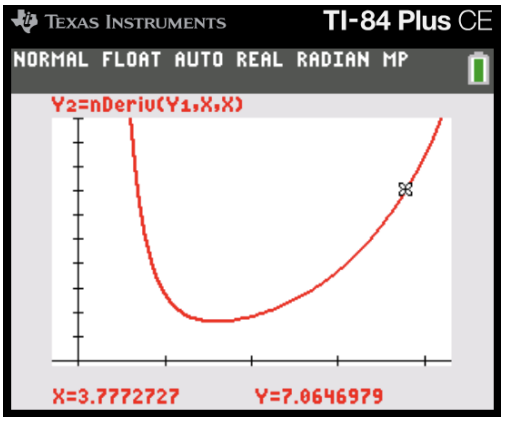
$$v(t) = \frac{dx}{dt} = \frac{e^t(t) - e^t(1)}{t^2} = \boxed{\frac{e^t(t-1)}{t^2}}$$

3) At what time is the particle *at rest* ?

$(t-1) = 0$ at $\boxed{t = 1\text{s}}$

4) Using your calculator, draw the graph of the *acceleration* of the parcicle for t between 1 and 4 sec.





5) Looking at this graph, evaluate *at what time* the *acceleration* is equal to 7 ms^{-2}.

Near 3.77 s