IB$_2$ **Test 3** Friday 24 Jan 2025

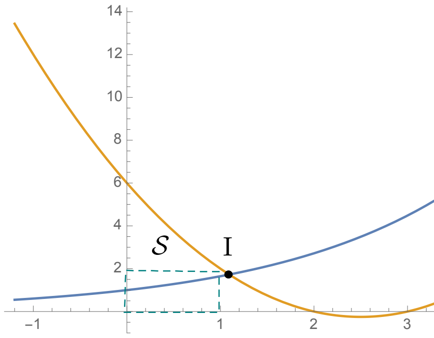
Subject : *Antiderivatives & Integrals*

Tot: /28 ANSWERS

Problem 1

The picture show the graph of the functions

$f(x)=e^{\frac{x}{2}}$ and $g(x)=x^2 \quad 5x+6$ The intercept I is at $\boxed{x=1.0937}$



i) The *area* of the surface $\mathcal{S}$ between the two curves and the y axis is

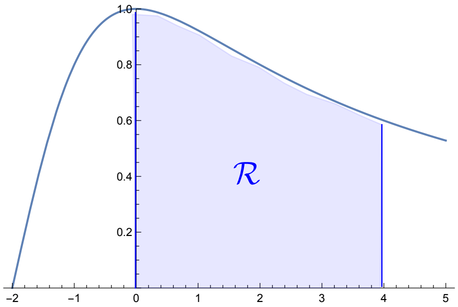
$$\int_0^{1.0937} e^{\frac{x}{2}}dx+\int_{1.0937}^{2}(x^2 \quad 5x+6)dx=\left[2e^{\frac{x}{2}}\right]_0^{1.0937}+\left[\frac{x^3}{3} \quad 5\frac{x^2}{2}+6x\right]_{1.0937}^{2}$$

$$=1.4556+0.652627=\boxed{2.10823}$$

ii) Area of the rectangle (size $x=1$ & $y=2$) is $1\times 2=2$, .
that is about 95% of area of $\mathcal{S}$.

Problem 2

The picture on the left shows the a part of the curve of the function $y=\frac{4x+8}{x^2+4x+8}$, for -2$\leqslant x \leqslant 5$



(a) Find x and y intercepts of the curve

(b) Show the region $\mathcal{R}$ that correspond to

$$\int_0^4 \frac{4x+8}{x^2+4x+8}dx$$

(c) Find the area of $\mathcal{R}$

paper 1 (without calculator)

$$\mathcal{R} = [2\ln(x^2+4x+8)]_0^4 = 2(\ln(40) \quad \ln(8))=\boxed{2\ln(5)}$$

paper 1 (with calculator)

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∫₀⁴((4*X+8)/(X²+4*X+8))dX
                3.218875825
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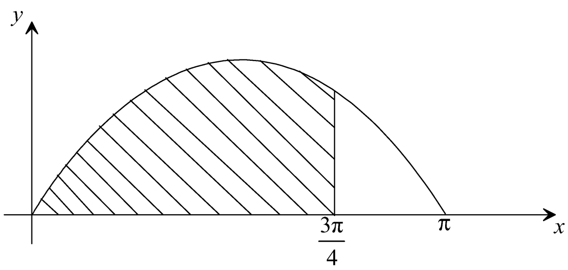
Problem 3

The diagram shows part of the curve $y=\sin x$. The shaded region is bounded by the curve and the lines $y=0$ and $x=\frac{3\pi}{4}$.



Given that $\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$ and $\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

$$\mathcal{A}=\int_0^{\frac{3\pi}{4}}\sin(x)dx$$

$$= \quad \cos(x)\Big|\Big|_0^{\frac{3\pi}{4}}$$

$$= \quad \cos\left(\frac{3\pi}{4}\right)+\cos(0)$$

$$= \boxed{1+\frac{\sqrt{2}}{2}}$$