Test 2

November 20. 2024

Maths IB_2

subjets : *Maximum and Minimum* & Optimisation,

Tot :[/ 28 marks]

Name: _____

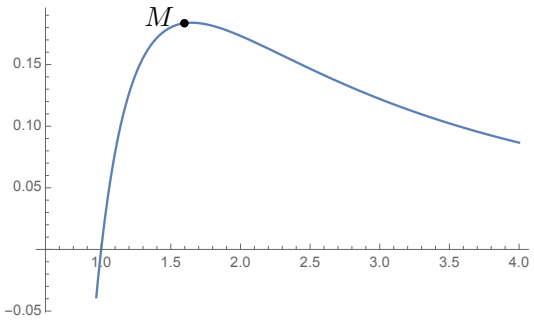
Problem 1

[/ 6 marks]

Let $g(x) = \frac{\ln(x)}{x^2}$

1. Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln(x)}{x^3}$

2. The figure below shows the graph of g.

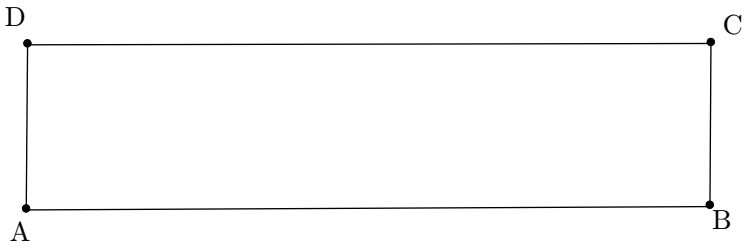


Find the coordinates of M.

Problem 2

[/ 9 marks]

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m2, as shown below.

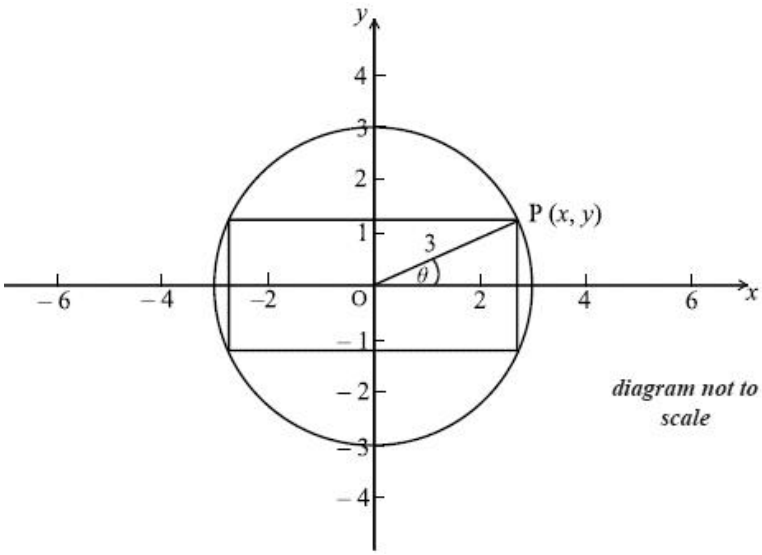


The fencing used for side AB costs 11€ per metre. The fencing for the other three sides costs 3€ per metre. The farmer creates an enclosure so that the cost is a minimum.
Find this minimum cost.

Problem 3

[/ 13 marks]

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x, y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

(a) Write down an expression in terms of θ for

(i) x;

(ii) y.

(2)

Let the area of the rectangle be A.

(b) Show that $A = 18 \sin 2\theta$.

(3)

(c) (i) Find $\frac{dA}{d\theta}$.

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

(8)

(Total 13 marks)