Maths IB_2

Test 1

9 October 2024

`subjets` : *First Principle & rules of derivation*

Answers

Tot :[/ 45 marks]

Problem 1 [5 marks]

Let us consider the function $f\colon x \mapsto x^2 \ \ 2x$

Using the *First Principle*: $f'(x_0) = \lim\limits_{h\to 0} \frac{(x^2 \ \ 2x) \ \ (x_0^2 \ \ 2x_0)}{x \ \ x_0} = \lim\limits_{h\to 0} \frac{(x^2 \ \ x_0^2) \ \ (2x \ \ 2x_0)}{x \ \ x_0}$
$\lim\limits_{h\to 0} \frac{(x \ \ x_0)(x+x_0) \ \ 2(x \ \ x_0)}{x \ \ x_0} = \lim\limits_{h\to 0} (x+x_0) \ \ 2 = 2x_0 \ \ 2$

The value of x_0 such that the *gradient* of the *tangent* at x_0 is 10 is $\boxed{x_0 = 6}$

Problem 2 [40 marks]

For the following functions, find

i) the *derivative*,

ii) the *gradient* of the *tangent* at $x = x_0$,

#	**function**	x_0	**derivative**	**gradient tangent at $x = x_0$**
1	$\frac{4}{5}x^5 \ \ 14x$	2	$4x^4 \ \ 14$	50
2	$36\cdot\sqrt[3]{x} = 36\cdot x^{\frac{1}{3}}$	8	$12\cdot x^{\ \frac{2}{3}} = \frac{12}{\sqrt[3]{x^2}}$	3
3	$8x + \frac{32}{\sqrt{x}} = 8x + 32x^{\frac{1}{2}}$	4	$8 + \frac{16}{\sqrt{x}}$	16
4	$7x \ \ \frac{3\pi}{2})\cos(x)$	$\frac{\pi}{2}$	$7\cos(x) \ \ 7x \ \ \frac{3\pi}{2})\sin(x)$	$0 \ \ 7\frac{\pi}{2} \ \ \frac{3\pi}{2}) = 2\pi$
5	$e^x\cos(x)$	$\frac{\pi}{4}$	$e^x(\cos(x) \ \ \sin(x))$	$e^x\left(\frac{\sqrt{2}}{2} \ \ \frac{\sqrt{2}}{2}\right) = 0$
6	$\frac{x^2 \ \ 1}{x^2+1}$	1	$\frac{4x}{(x^2+1)^2}$	1
7	$e^{\sin(x)}$	0	$e^{\sin(x)}\cos(x)$	1
8	$x^2\ln(x)$	1	$2x\ln(x) + x^2\frac{1}{x} = x(2\ln(x)+1)$	1
9	$\cos^2(x)$	$\frac{\pi}{4}$	$2\cos(x)\sin(x)$	$2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = 1$
10	$\sin(x^3)$	$\sqrt[3]{\pi}$	$3x^2\cos(x^3)$	$3 \ \sqrt[3]{\pi})^2\cos(\pi) = \ \ 3\sqrt[3]{\pi^2}$

Bonus The derivative of $f(x) = e^{(e^{(e^x)})}$ is $\boxed{e^{(e^{(e^x)})}\cdot e^{(e^x)}\cdot e^x}$ [+3]