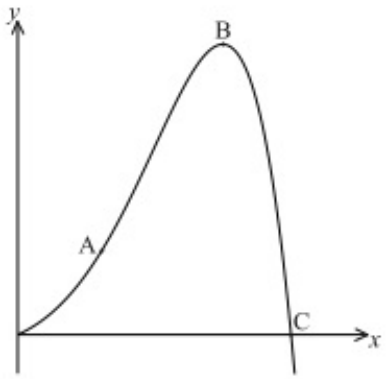
Problem 1 ***[5 marks]***

Find $\int_{\frac{5}{3}}^{\frac{\pi}{3}-\frac{5}{3}} [2\cos(3x+5)+3]\,dx$

Problem 2 ***[15 marks]***

The function f is defined as $f(x)=e^x \sin x$, where x is in radians. Part of the curve of f is shown below.



There is a point of inflexion at A, and a local maximum point at B. The curve of f intersects the x-axis at the point C.

(a) Write down the x-coordinate of the point C. *[1 mark]*

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B. *[4 marks]*

(c) Show that $f''(x)=2e^x \cos x$. *[2 marks]*

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A. *[4 marks]*

(e) Let R be the region enclosed by the curve and the x-axis, between the origin and C.

(i) Write down an expression for the area of R.

(ii) Find the area of R. *[4 marks]*

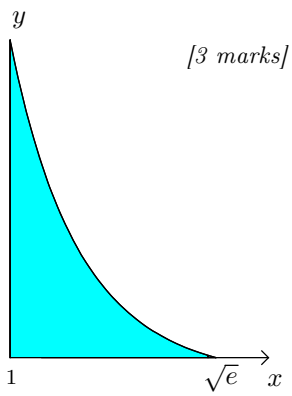
Problem 3 *[10 marks]*

Let $g(x)=\dfrac{\ln x}{x^2}$, for $x>0$.

(a) Use the quotient rule to show that $g'(x)=\dfrac{1-2\ln x}{x^3}$. *[4 marks]*

(b) The graph of g has a maximum point at A. Find the x-coordinate of A. *[3 marks]*

(c) The picture on the left shows the a part of the curve of function $y=\frac{1}{x^3}\frac{2\ln(x)}{}$, for x between 1 and $\sqrt{e}$. Find the surface area enclosed by the this curve and th x axis (x between 1 and $\sqrt{e}$). *[3 marks]*



Bonus *[+5]*

i) Let $f'(x)=3x^2+2$

Given that $f(2)=6$, find $f(x)$

ii) Let $g'(x)=\cos(2x)$

Given that $f(\frac{\pi}{4})=\frac{7}{2}$, find $g(x)$

iii) Show that $(f\circ g)(0)=3^3$

[5 marks]