**Defition :** Newton's *first principle* : $f'(x) = \lim_{h\to 0} \frac{f(x+h) \quad f(x)}{h}$  Leibniz : $f'(x) = \frac{df}{dx}$

**Interpretation** : *Instantaneous rate of change* and *grandient of the tangent* line to the curve of $f$.

**Rules**:

| # | used for | formula | Leibniz notation IB booklet |
|---|---|---|---|
| 1 | *multiplic.by a scalar* $(\lambda \in \mathbb{R})$ | $(\lambda f(x))' = \lambda f'(x)$ | $\frac{d}{\mathrm{dx}}(\lambda y) = \lambda \frac{\mathrm{dy}}{\mathrm{dx}}$ |
| 2 | *sum* or *difference* | $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ | $\frac{d}{\mathrm{dx}}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$ |
| 3 | *product* | $(f(x)\cdot g(x))' = f'(x)g(x) + f(x)g'(x)$ | $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ p.6 |
| 4 | *ratio* | $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) \quad f(x)g'(x)}{g^2(x)}$ | $\frac{d}{dx}\ \ \frac{u}{v}) = \frac{u\frac{dv}{dx} \quad v\frac{du}{dx}}{v^2}$ p.6 |
| 5 | *composition* : $y = g(f(x))$ | $y' = f'(g(x))\cdot g'(x)$ | $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ p.6 |

| 6 | $f(x) = x^n$ | $f'(x) = nx^{n \quad 1}$ |
|---|---|---|

can be used *not only* for $n \in \mathbb{N}$ like 0,1,2,3,...
but *also* for $n \in \mathbb{Q}$, like $\frac{1}{2}$ , $\frac{1}{3}$, $\frac{7}{3}$, $\ \frac{1}{2}$, $\ \frac{3}{4}$ etc....

**Examples**

**i)** $(3x)' = 3x' = 3$  by rules 1 and 6  notice: $x = x^1$ then $x' = 1 \quad x^0 = 1$

**ii)** $\ 4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2)' = 12x^2 \quad 3x + 6$  by rules 1, 2 and 6.  notice: $(1)' = (x^0)' = 0$

**iii)** $\ 4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2) \times \sqrt{x}\ )'$

$= \ 4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2)' \times \sqrt{x} + \ 4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2) \times (\sqrt{x})'$  by rule 3

$= (12x^2 \quad 3x + 6)\sqrt{x} + \ 4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2)\frac{1}{2\sqrt{x}}$

$= \frac{(12x^2 \quad 3x+6)\,2x + \left(4x^3 \quad \frac{3}{2}x^2 + 6x \quad 2\right)}{2\sqrt{x}} = \frac{28x^3 \quad \frac{15}{2}x^2 + 18x \quad 2}{2\sqrt{x}}$

**iv)** $\left(\frac{12x^2 \quad 3x+6}{\sqrt{x}+1}\right)' = \frac{(12x^2 \quad 3x+6)' \times (\sqrt{x}+1) \quad (12x^2 \quad 3x+6) \times (\sqrt{x}+1)'}{(\sqrt{x}+1)^2}$  by rule 4

$= \frac{(24x \quad 3) \times (\sqrt{x}+1) + \ (12x^2 \quad 3x+6) \times \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2}$  by rules 1, 2 and 6

**v)** $\left(\sqrt{(x^2+4x+7)}\right)' = \frac{1}{2\sqrt{(x^2+4x+7)}} \times (2x+4)$  by rule 5  ... $= \frac{x+2}{\sqrt{(x^2+4x+7)}}$

**vi)** $\left(\sqrt[3]{(x^2+x \quad 2)^2}\right)' = ((x^2+x \quad 2)^{2/3})' = \frac{2}{3}(x^2+x \quad 2)^{\frac{2}{3} \quad 1} \times (x^2+x \quad 2)'$  again by rule 5

$= \frac{2}{3}(x^2+x \quad 2)^{\ \frac{1}{3}} \times (2x+1)$  by rules 1 and 2

$= \frac{2(2x+1)}{3 \times \sqrt[3]{x^2+x \quad 2}}$

**vii)** $[((1+x^2)^3+7)^4]' = 4((1+x^2)^3+7)^3 \times ((1+x^2)^3+7)' = 4((1+x^2)^3+7)^3 \times 3(1+x^2)^2 \times 2x$

using rule 5 two times !

**viii)** $\left[\frac{(x^7+1)^5}{3x^2}\right]' = \frac{5(x^7+1)^4\,(7x)(3x^2) \quad (x^7+1)^5(6x)}{9x^4}$  mainly by applaying rules 4 & 5