Maths IB_2 HL

Exercises

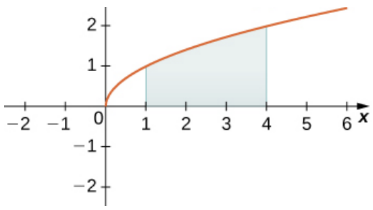
Subject : *Volume of a solid of revolution*

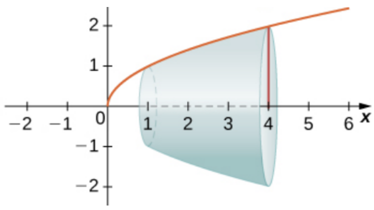
Wednesday 8.10.2025

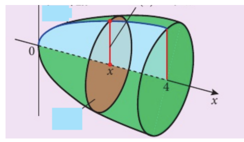
ANSWERS

I Theory

<u>First method</u> (around the $\boldsymbol{x}$ **– axis**)





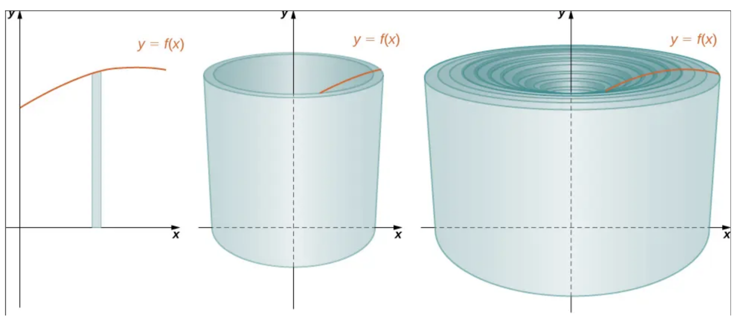


Formula : $$\text{V}_{\text{aroud } x-\text{axis}} = \pi \int_{x_1}^{x_2} (f(x))^2 dx$$

For the rotation around the **y-axis**, we can use the inverse function (that is $f^{-1}(x)$)

Then the formula becomes = $$\text{V}_{\text{aroud } y-\text{axis}} = \pi \int_{y_1}^{y_2} (f^{-1}(y))^2 dx$$

<u>Second method</u> : (around the $\boldsymbol{y}$ **– axis**)

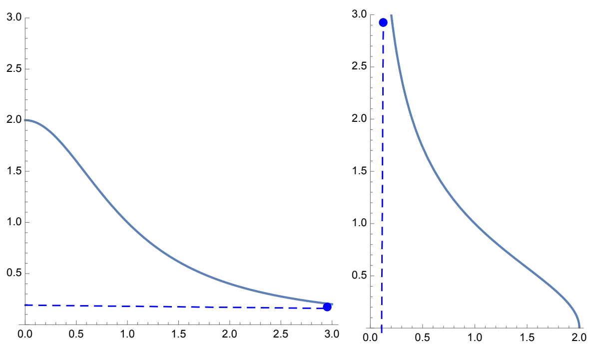


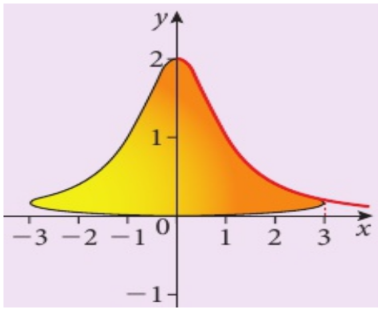
Formula : $$\text{V}_{\text{aroud } y-\text{axis}} = 2\pi \int_{x_1}^{x_2} x f(x) dx$$

II Exercise

1) Let us consider the curve of equation $y = f(x) = \frac{2}{x^2+1}$

The first figure below shows this curve, and the curve of $y = f^{-1}(x)$





second figure

The second figure show S the solid of revolution of the curve $y = f(x)$ around the $\boldsymbol{y}$ **− axis**, for x between 0 and 3.

1] Calculate the volume V of S of the solid generated by the rotation around the $\boldsymbol{x}$ **− axis**

i) Using the first method, (with the inverse function: as $y = \frac{2}{x^2+1}$, we have $y = \sqrt{\frac{1}{y} - 1}$)

$$V_{\text{aroud } y-\text{axis}} = \pi \int_{y_1}^{y_2} (f^{-1}(y))^2 dx = \pi \int_{\frac{2}{10}}^{2} \left(\sqrt{\frac{1}{y} - 1}\right)^2 dx = \pi \int_{\frac{2}{10}}^{2} \left(\frac{1}{y} - 1\right) dx$$

$$= \pi[\ln(y) - y]_{\frac{2}{10}}^{2} = \pi\left[(\ln(2) - 2) - \left(\ln(2) - \ln(10) - \frac{2}{10}\right)\right]$$

$$= \pi\left[\left(\ln(10) - \frac{9}{5}\right)\right] = \boxed{\pi \ln(10) - \frac{9}{5}\pi}$$

ii) Using the second method.

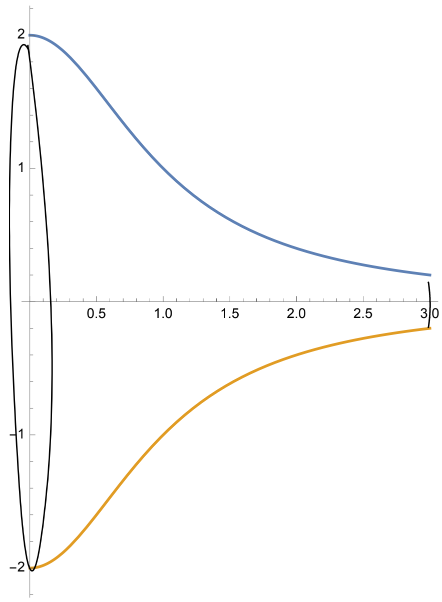
$$V_{\text{aroud } y-\text{axis}} = 2\pi \int_{x_1}^{x_2} x f(x) dx = 2\pi \int_0^3 x f(x) dx = \pi \int_0^3 \frac{2x}{x^2+1} dx = \pi[\ln(x^2+1)]_0^3$$

$$= \pi[\ln(9+1) - \ln(1)] = \boxed{\pi \ln(10)}$$

Question : How to explain the difference of $\frac{9}{5}\pi$ between the two results ?

Which one is the correct result ?

2] The volume of the solid of revolution generated by the rotation of the curve $y = f(x)$ around the x axis, for x between 0 and 3



is :

$$\mathrm{V}_{\text{aroud } x-\text{axis}} = \pi \int_{x_1}^{x_2} (f(x))^2 dx$$

$$= \pi \int_0^3 \left(\frac{2}{x^2+1} \right)^2 dx$$

$$= 2\pi \left[\frac{x}{x^2+1} + \arctan(x) \right]_0^3$$

$$= \boxed{\frac{6\pi}{5} + \arctan(5)}$$