Maths IB_2 HL

Exercises

Subjects : *Implicit derivatives*

Thursday 16.9.2026

Name:____________

Question 1

A curve is given by the equation $y = \dfrac{e^{2x}-1}{e^{2x}+1}$, $x \in \mathbb{R}$.

(a) By applying l'Hôpital's rule or otherwise, show that $\lim_{x\to\infty}\left(\dfrac{e^{2x}-1}{e^{2x}+1}\right) = 1$. [2]

(b) (i) Show that $\dfrac{dy}{dx} = \dfrac{4e^{2x}}{\left(e^{2x}+1\right)^2}$.

(ii) Hence, show that $1 - y^2 = \dfrac{dy}{dx}$. [6]

(c) (i) By using implicit differentiation and the result in part (b)(ii), show that $\dfrac{d^2y}{dx^2} = 2y^3 - 2y$.

(ii) Hence, find an expression for $\dfrac{d^3y}{dx^3}$ in terms of y. [5]

(d) By using your results from parts (b) and (c), find the Maclaurin series for $\dfrac{e^{2x}-1}{e^{2x}+1}$ up to and including the term in x^3. [4]

Question 2

A curve is given by the equation $y = \dfrac{e^{2x}-1}{e^{2x}+1}$, $x \in \mathbb{R}$.

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