Maths IB_2 HL

Exercises

Subject : *Maclaurin series for composite functions*

Wednesday 8.10.2025

Example

We know : $\ln(1+x)= x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+O(x^7)$ [1]

and $\sin(x)=x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+O(x^9)$ [2]

we would like to show that $\ln(1+\sin(x))=x-\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{12}+O(x^5)$

For doing that, we will insert the first terms of [2] in the first terms of [1]

$$\ln(1+\sin(x))=\left(x-\frac{x^3}{6}\right)-\frac{\left(x-\frac{x^3}{6}\right)^2}{2}+\frac{\left(x-\frac{x^3}{6}\right)^3}{3}-\frac{\left(x-\frac{x^3}{6}\right)^4}{4}+O(x^5)$$

$$= x-\frac{x^3}{6}-\frac{\frac{x^6}{36}-\frac{x^4}{3}+x^2}{2}+\frac{x^3-\frac{x^5}{2}+\dots}{3}-\frac{x^4-\frac{2x^6}{3}+\cdots+}{4}+O(x^5)$$

$$=x-\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{12}+O(x^5) \qquad \text{ok}$$

Exercises

1) Show that

$$\ln(1+\cos(x))=\log(2)-\frac{x^2}{4}-\frac{x^4}{96}+O(x^5)$$

and $\ln(\cos(x))=-\frac{x^2}{2}-\frac{x^4}{12}-\frac{x^6}{45}+O(x^7)$

2) Show that $e^{(e^x-1)}=1+x+x^2+\frac{5x^3}{6}+\frac{5x^4}{8}+O(x^5)$

3) What gives $\ln(e^x)$?

hint: $x+\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^2+x^3+\cdots+}{2}+\frac{x^3+\cdots+}{3}$