Maths IB_2 AA HL

Test 3

Subjects : *Integration, area...*

Total : / 30

Thursday 11.11.2025

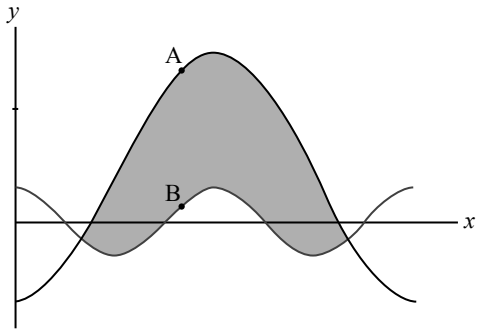
Name: _____

Problem 1

[max / 17 parks]

[Maximum mark: 17]

Consider the functions f and g defined on the domain $0 < x < 2\pi$ by $f(x) = 3\cos 2x$ and $g(x) = 4 - 11\cos x$.
The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$.



(a) Find the x-coordinates of the points of intersection of the two graphs.

(b) Find the exact area of the shaded region, giving your answer in the form $p\pi + q\sqrt{3}$, where $p, q \in \mathbb{Q}$.

At the points A and B on the diagram, the gradients of the two graphs are equal.

(c) Determine the y-coordinate of A on the graph of g.

Problem 2

[max / 8 parks]

A tank of water initially contains 400 litres. Water is leaking from the tank such that after 10 minutes there are 324 litres remaining in the tank.

The volume of water, V litres, remaining in the tank after t minutes, can be modelled by the differential equation

$$\frac{dV}{dt} = -k\sqrt{V}, \text{ where } k \text{ is a constant.}$$

(a) Show that $V = \left(20 - \frac{t}{5}\right)^2$. [6]

(b) Find the time taken for the tank to empty. [2]

Problem 3

[max / 5 parks]

(a) Integrate $\int \frac{\sin\theta}{1-\cos\theta} d\theta$. *[3 marks]*

(b) Given that $\int_{\frac{\pi}{2}}^{a} \frac{\sin\theta}{1-\cos\theta} d\theta = \frac{1}{2}$ and $\frac{\pi}{2} < a < \pi$, find the value of a. *[2 marks]*