Maths IB_2AA HL

Test 2

Subjects : *Limits, MacLaurin & integration*

Total : / 21

Thursday 4.11.2025

IB Marksheme

IB Nov 2023

(a) attempt to use chain rule to find $f'(x)$ ***(M1)***

$f'(x)=(-2\sin 2x)e^{\cos 2x}(=0)$ ***A1***

$\Rightarrow \sin 2x = 0$ ***(M1)***

$2x = 0, \pi, 2\pi, \ldots$

$x = 0, \dfrac{\pi}{2}, \pi, \ldots$ ***A1***

Coordinates are $(0, e), \left(\dfrac{\pi}{2}, \dfrac{1}{e}\right), (\pi, e)$ ***A1***

Note: Special case: For two correct coordinate pairs award ***(M1)A1(M1)A0A1.*** For extra coordinate pairs award ***(M1)A1(M1)A1A0.***

[5 marks]

(b) attempt to differentiate $f'(x)$ using product rule ***(M1)***

$f''(x)=(-2\sin 2x)(-2\sin 2x)e^{\cos 2x}-(4\cos 2x)e^{\cos 2x}$ ***A1***

at $x=0$, $f''(x)=-4e<0$ so maximum **AND**

at $x=\pi$, $f''(x)=-4e<0$ so maximum ***R1***

at $x=\dfrac{\pi}{2}$, $f''(x)=\dfrac{4}{e}>0$ so minimum ***R1***

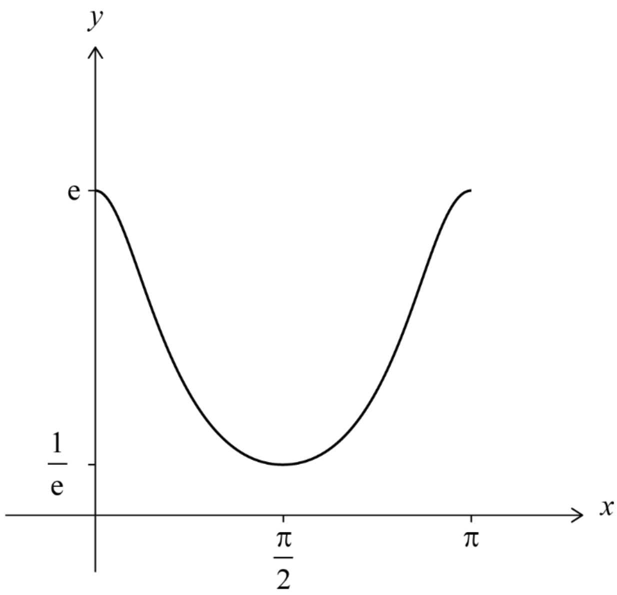
Note: The values for the second derivative must be correct in order to award the ***R*** marks.

[4 marks]

continued…

Question continued

(c)



A1A1A1

Note:	Award ***A1*** for general shape, ***A1*** for correct maxima $(0, \mathrm{e}), (\pi, \mathrm{e})$ and minimum point $\left(\frac{\pi}{2}, \frac{1}{\mathrm{e}}\right)$ and ***A1*** for showing a higher rate of change of gradient at maxima and a lower rate of change of gradient at the minimum point.

[3 marks]

continued…

Question continued

(d) (i) $\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \cdots$ ***(M1)***

$\cos 2x = 1 - 2x^2 + \frac{2x^4}{3} \cdots$ ***A1***

(ii) **METHOD 1**

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots$

attempt to substitute series for $\cos 2x - 1$ into series for e^x ***(M1)***

Note: Award ***(M0)*** for substituting the Maclaurin series for $\cos 2x$ into the Maclaurin series for e^x.

$e^{\cos 2x - 1} = 1 + \left(-2x^2 + \frac{2x^4}{3}\right) + \frac{\left(-2x^2 + \frac{2x^4}{3}\right)^2}{2} + \ldots$ ***A1***

$\left(= 1 - 2x^2 + \frac{2x^4}{3} + 2x^4 + \ldots\right)$

$= 1 - 2x^2 + \frac{8x^4}{3} + \ldots$ ***A1***

METHOD 2

$e^{\cos 2x - 1} = e^{-2x^2 + \frac{2x^4}{3}} = e^{-2x^2} e^{\frac{2x^4}{3}}$

attempt to find the Maclaurin series for e^{-2x^2} OR $e^{\frac{2x^4}{3}}$ ***(M1)***

$e^{-2x^2} = 1 - 2x^2 + 2x^4 + \ldots$; $e^{\frac{2x^4}{3}} = 1 + \frac{2}{3}x^4 + \ldots$

$e^{-2x^2} e^{\frac{2}{3}x^4} = \left(1 - 2x^2 + 2x^4 + \ldots\right)\left(1 + \frac{2}{3}x^4 + \ldots\right)$ ***A1***

$= 1 - 2x^2 + \frac{8x^4}{3} + \ldots$ ***A1***

continued…

Question continued

(iii) $\left(f\left(x\right)\approx\right)\mathrm{e}\left[1-2x^2+\frac{8x^4}{3}+\ldots\right]\left(=\mathrm{e}-2\mathrm{e}x^2+\frac{8\mathrm{e}x^4}{3}+\ldots\right)$ ***A1***

[6 marks]

(e) $\int_0^{\frac{1}{10}}\mathrm{e}^{\cos 2x}\,\mathrm{d}x\approx\mathrm{e}\int_0^{\frac{1}{10}}\left(1-2x^2\right)\mathrm{d}x$ ***(M1)***

$=\mathrm{e}\left[x-\frac{2x^3}{3}\right]_0^{1/10}$ ***A1***

$=\mathrm{e}\left(\frac{1}{10}-\frac{2}{3000}\right)$ ***A1***

$=\frac{298\mathrm{e}}{3000}$

$=\frac{149\mathrm{e}}{1500}$ ***AG***

Note: If candidate follows through an incorrect expansion from part (d), award a maximum of ***M1A1FTA0***

[3 marks]

Total [21 marks]