

IB Nov 2023

Consider the function $f(x) = e^{\cos 2x}$, where $-\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$.

- (a) Find the coordinates of the points on the curve $y = f(x)$ where the gradient is zero. [5]
- (b) Using the second derivative at each point found in part (a), show that the curve $y = f(x)$ has two local maximum points and one local minimum point. [4]
- (c) Sketch the curve of $y = f(x)$ for $0 \leq x \leq \pi$, taking into consideration the relative values of the second derivative found in part (b). [3]
- (d) (i) Find the Maclaurin series for $\cos 2x$, up to and including the term in x^4 .
- (ii) Hence, find the Maclaurin series for $e^{\cos 2x - 1}$, up to and including the term in x^4 .
- (iii) Hence, write down the Maclaurin series for $f(x)$, up to and including the term in x^4 . [6]
- (e) Use the first two non-zero terms in the Maclaurin series for $f(x)$ to show that
- $$\int_0^{1/10} e^{\cos 2x} dx \approx \frac{149e}{1500}. \quad [3]$$