

Question 1

Find the *domain* of the following functions

$$1) f_1(x) = \frac{31x^5 - 17\left(1 + \frac{2}{3}\right)^8 + x(x+1)(x+2) - 22}{17}$$

$$D_1 = \mathbb{R}$$

$$2) f_2(x) = \frac{(x-1)(x+2)(x-3)}{\sqrt[3]{x}(x+1)^2}$$

$$D_2 = \mathbb{R} \setminus \{-1, 0\}$$

$$3) f_3(x) = \frac{\sqrt{x-1}}{(x-2)(x+3)}$$

$$D_3 = [1, \infty[\setminus \{-3, 2\} = [1, \infty[\setminus \{2\}$$

$$4) f_4(x) = \frac{\sqrt[4]{\frac{x}{2} - 4}}{\sqrt[6]{9-x}}$$

$$D_4 = [8, 9[$$

$$5) f_5(x) = \frac{13\sqrt{5-x}}{x} - \frac{18}{5} \frac{\log_5(x-2)}{x-1}$$

$$D_5 =]2, 5]$$

Question 2

Given $f(x) = \frac{4x}{2x+1}$ and $g(x) = \frac{1-x}{2}$

$$1) (f \circ g)(x) = \frac{4 \frac{1-x}{2}}{2 \frac{1-x}{2} + 1} = \frac{2(1-x)}{1-x+1} = \frac{2(1-x)}{2-x}$$

$$2) (f \cdot g)(x) = \frac{4x}{2x+1} \times \frac{1-x}{2} = \frac{2x(1-x)}{2x+1}$$

$$3) (f \circ g)(x) = (f \cdot g)(x) \Leftrightarrow \frac{2(1-x)}{2-x} = \frac{2x(1-x)}{2x+1} \Leftrightarrow x=1 \quad \text{or} \quad \frac{1}{2-x} = \frac{x}{2x+1}$$

$$\Leftrightarrow 2x+1 = x(2-x) \Leftrightarrow x^2 = -1 \quad x \notin \mathbb{R}$$

$$\text{then } S = \{1\}$$