Mathematics - IB_1 -

Test 4
subject: *Trigonometry I*

Wednesday 5 fev 2025

Total : / 36 marks

ANSWERS

Problem 1 *[12 marks]*

Assuming θ is in the *third* sector, and $\cos(\theta)=-\sqrt{\frac{4}{7}}$, give the exact expression for

i) $\sin(\theta)$, **ii)** $\tan(\theta)$ **iii)** $\cos(2\theta)$ **iv)** $\sin(2\theta)$, **v)** $\tan(2\theta)$

i) $\boxed{-\sqrt{\frac{3}{7}}}$ **ii)** $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$ **iii)** $\left(-\sqrt{\frac{4}{7}}\right)^2-\left(-\sqrt{\frac{3}{7}}\right)^2=\boxed{\frac{1}{7}}$ **iv)** $2\left(-\sqrt{\frac{3}{7}}\right)\left(-\sqrt{\frac{4}{7}}\right)=\boxed{\frac{4\sqrt{3}}{7}}$ **v)** $\boxed{4\sqrt{3}}$

Problem 2 (without calculator) *[13 marks]*

Solve the following trigonometric equations:

1) $\sin(3x)=\frac{\sqrt{3}}{2}$ *for* $0\leqslant x<360^0$ *[4 marks]*

As $\frac{\sqrt{3}}{2}>0$, then angle $3x$ is either in the *first* or in the *second* sector.

In the *first* : $3x=60+k360$ $\Rightarrow$ $\boxed{x=20+k120}$ in the *second* : $3x=120+k360$ $\boxed{x=40+k120}$

that means $\boxed{20^0, 40^0, 140^0, 160^0, 260^0 \text{ and } 280^0}$ are *six distinct solutions* between 0 and 360^0

2) $\cos(4x)=\frac{\sqrt{2}}{2}$ *[5 marks]*

$4x=\frac{\pi}{4}$ $(+2k\pi)$ or $4x=\frac{7\pi}{4}$ $(+2k\pi)$

$\Rightarrow x=\boxed{\frac{\pi}{16}+k\frac{\pi}{2}}$ or $x=\boxed{\frac{7\pi}{16}+k\frac{\pi}{2}}$

that means $S=\left\{\frac{\pi}{16},\frac{7\pi}{16},\frac{9\pi}{16},\frac{15\pi}{16},\frac{17\pi}{16},\frac{23\pi}{16},\frac{25\pi}{16},\frac{31\pi}{16}\right\}$ are *height distinct solutions* between 0 and 2π

3) $6\sin x-4\cos^2 x=0$, $0\leqslant x<3\pi$ *(radian)* *[5 marks]*

$6\sin x-4\left(1-\sin(x)^2\right)=0 \Rightarrow 4\sin(x)^2+6\sin(x)-4=0$

then $\sin(x)=\frac{-6\mp\sqrt{100}}{8}=\begin{cases}-2\\ \frac{1}{2}\end{cases}$ (notice $\cos(x)$ *cannot* be equal to -2)

$\Rightarrow \sin(x)=\frac{1}{2} \Rightarrow \boxed{x=\frac{\pi}{6} \text{ or } x=\frac{5\pi}{6} \text{ or } x=\frac{13\pi}{6} \text{ or } x=\frac{17\pi}{6}\,(\text{rad})}$ assuming $0\leqslant x<3\pi$

Problem 3 (with or without calculator) *[9 marks]*

Consider the *trigonometric equation* : $5\cos(2\theta)+3\cos(\theta)-2=-3$

i) Show it can be written as $a\cos^2(\theta)+b\cos(\theta)+c=0$ (find a, b, c)

$5(\cos^2(\theta)-(1-\cos^2(\theta)))+3\cos(\theta)-2=-3 \Rightarrow \boxed{10\cos^2(\theta)+3\cos(\theta)-4=0} \Rightarrow \begin{matrix}a=10\\ b=3\\ c=-4\end{matrix}$

ii) $\Delta=169$ $\cos(\theta)=\frac{-3\pm13}{20}=\frac{1}{2}$ or $-\frac{4}{5}$

$\Rightarrow \boxed{\theta\in\left\{\frac{\pi}{3}\text{rad}\ \frac{5\pi}{3}\text{rad}\right\}\cup\{2.498\,\text{rad}, 3.785\text{rad}\}}$