Test 3

Wednesday 20.11.2024

Maths AA IB$_1$ *Arithmetic or Geometric sequences & Series*

Total : / 24

ANSWERS

Question 1 **[4 marks]**

An arithmetic sequence has a first term of 20 and a common difference of 13. $u = 20 + (n \quad 1)13 = 7 + 13n$

Identify which term is equal to 228. Find the sum of all the terms 20, 33, ..., 228. $228 = 7 + 13n \Rightarrow n = \frac{221}{13} = \boxed{17}$

The sum is $s_{17} = \frac{17}{2}(2 \cdot 20 + 16 \times 13) = \frac{17}{2}248 = \boxed{2108}$

Question 2 **[5 marks]**

Consider the sequence, $S_n = 2n^2 - 3n$.

$s_1 = \quad 1 = u_1$

$s_1 = 2 = u_1 + u_2 \Rightarrow u_2 = 3 \qquad d = u_2 \quad u_1 = 4$

Find an expression for the general term, u_n, and show that this is an arithmetic sequence.

$u_n = u_1 + (n \quad 1)s$

$u_n = \quad 1 + (n \quad 1)4 = \boxed{4n \quad 5}$

Question 3 **[4 marks]**

A geometric sequence has a second term of 14 and a sixth term of 224.

$14 = u_1 r^{2 \quad 1}$

$224 = u_1 r^{6 \quad 1}$

by division : $16 = \quad r^4 \Rightarrow r \pm \sqrt[4]{16} = \boxed{\pm 2}$

Find the possible values of the common ratio and the first term.

$u_1 = 14 \div (\pm 2) = \boxed{\pm 7}$

Question 4 **[5 marks]**

(a) Find the number of terms in the geometric series

$1+3+9+27+\ldots \ +177147$

$177147 = 3^{(n \quad 1)} \Rightarrow n \quad 1 = \log_3(177147) = 12 \quad \Rightarrow \boxed{n = 12}$

(b) Calculate the sum of the series in part (a)

$\boxed{s_{12} = 1\frac{1 \quad 3^{12}}{1 \quad 3} = 265720}$

Question 5 **[6 marks]**

Sam invests 1700€ in a savings account that pays a nominal annual rate of interest of 2.74 %, compounded *half-yearly**.

(a) Find the amount that Sam will have in his account after 10 years.

David also invests 1700€ in a savings account that pays an annual rate of interest of r% , compounded *yearly*. $1700\left(1 + \frac{2.74}{100 \times 2}\right)^{2 \times 12} = \boxed{2356.6€}$

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. $1776 = 1700 \ \ 1 + \frac{r}{100})^{12} \Rightarrow \boxed{r = 2.76\%}$

* *compuesto semestral* 半年ごとの複合