Maths HL $IB_1$ **Binomial Expansion** Tuesday 3.12.2024

## With *negative* exponent[1]

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We saw in maths SL the formula for the binomial expansion

$$\boxed{(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k\, b^{n-k}} \quad (1)$$

The expansion start with : $a^n + \frac{n}{1!}a^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + \ldots$ (2)
And works for any $n \in \mathbb{N}$.
We will see a formula that can be used for negative exponents, like $(a+b)^{-4}$
In IB books formula is given for $(1+x)^n$ but we saw in class how to use if for $(a+b)^n$ by transforming $(a+b)$ in $a(1+\frac{b}{a})$. Its suppose $|x| < 1$.
This formula is :

$$\boxed{(1+x)^n = 1 + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-2)}{4!}x^4 + \ldots} \quad (3)$$

You can observe: Its looks exactly like what we get taking $a = 1$ and $b = x$ in (1) !
But the is an important difference : This expansion never ends ( it continues to infinity ).
You can read more about it in the book maths Oxford maths for IB diploma AA HL p.63.
The following investigating is take for this book:

**1)** Show that the formula (2) can be written as

$$\boxed{(1-x)^{-n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k} \quad (4)$$

The table below includes the results obtained above.

| $(1-x)^i$ | Constant | Coefficient of $x$ | Coefficient of $x^2$ | Coefficient of $x^3$ | Coefficient of $x^4$ | Coefficient of $x^5$ |
|---|---|---|---|---|---|---|
| $(1-x)^{-1}$ | 1 | 1 | 1 | 1 | 1 | 1 ... |
| $(1-x)^{-2}$ | 1 | | 3 | 4 | 5 | 6... |
| $(1-x)^{-3}$ | 1 | −3 | +6 | −10... | | |
| $(1-x)^{-4}$ | | | | | | |
| $(1-x)^{-5}$ | | | | | | |

**2** Use Newton's generalization to verify that the coefficients shown in the table are correct.

**3** Apply Newton's generalization to copy and complete the table.

Can you find from (4) a formula for $(a+b)^n$ for negative integer $n$ (assuming $a > b$) ?

<u>Example of IB questions</u> (for more similar questions , see exercise 1J p.65 of you book)

**1)** Expand of $\frac{2}{1+2x}$ and of $\frac{2}{(1-x)^3}$

**2)** Show that $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64} + \cdots = \frac{1}{3}$

**3)** Show that $\frac{x}{(1+x)^2} = 1 - 2x + 3x^3 - 4x^4 + \ldots$

**4)** Show that $\frac{1}{1+\frac{i}{2}} = \frac{4}{5} + \frac{2}{5}\mathrm{i}$

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1. There is an other formula for expanding $(a+b)^n$ for $n \in \mathbb{Q}$. That will not be in your next exam :)