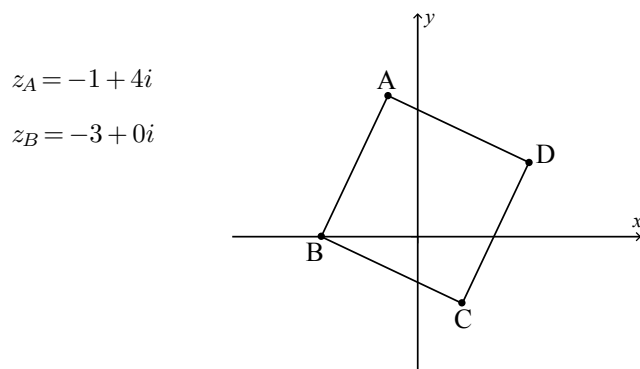


Question 1

[5 marks]

- In the following Argand diagram the point A represents the complex number $-1 + 4i$ and the point B represents the complex number $-3 + 0i$. The shape of ABCD is a square.
- i) Determine the complex numbers represented by the points C and D.



- ii) What complex number z_O represent the *center* of the square?

- iii) What is the product $(z_A - z_O) \cdot (z_A - z_O) \cdot (z_A - z_O) \cdot (z_D - z_O)$?

i) $\vec{AB} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ donc $\vec{AD} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \Rightarrow z_D = 3 + 2i$ et $z_C = 1 - 2i$

ii) $z_O = i$

iii) $z_A \cdot z_B \cdot z_C \cdot z_D = (i + 1 - 4i)(i + 3)(i - 3 - 2i)(i - 1 + 2i)$
 $= (1 - 3i)(3 + i)(-3 - i)(-1 + 3i)$
 $= (6 - 8i)(6 - 8i)$
 $= (100 - 96i) = 8(25 - 12i)$

Question 2

[5 marks]

Consider the complex numbers $z_1 = \cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}$ and $z_2 = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$.

(a) (i) Find $\frac{z_1}{z_2} = \frac{\text{cis}(\frac{11\pi}{12})}{\text{cis}(\frac{\pi}{6})} = \text{cis}\left(\frac{(11-2)\pi}{12}\right) = \text{cis}\left(\frac{9\pi}{12}\right) = \text{cis}\left(\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right) - i \sin\left(\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$

(ii) Find $\frac{z_2}{z_1} = \frac{\text{cis}(\frac{\pi}{6})}{\text{cis}(\frac{11\pi}{12})} = \text{cis}\left(\frac{(2-11)\pi}{12}\right) = \text{cis}\left(-\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i}$ [3]

- (b) 0 , $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$ are represented by three points O, A and B respectively on an Argand diagram. Determine the area of the triangle OAB. $\mathcal{A} = \frac{bh}{2} = \frac{(\frac{\sqrt{2}}{2}) \cdot (\frac{\sqrt{2}}{2})}{2} = \boxed{\frac{1}{2}}$ [2]

Question 3

$$\boxed{4096(\sqrt{3} - i)}$$

[5 marks]