MATHS AA SL IB$_1$

Easter Examinations

PAPER II

Friday 22 March 2024

Duration : 1h 30min

ANSWERS

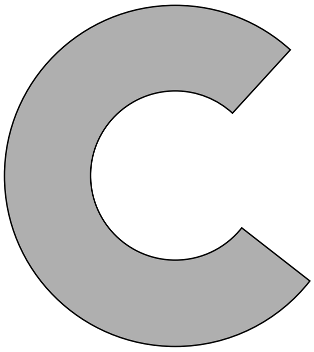
5 questions Total : / 50 marks

The use of a calculator is permitted for this paper

Problem 1

[7 marks]

A company is designing a new logo in the shape of a letter “C”.



The letter “C” is formed between two circles with centre O.

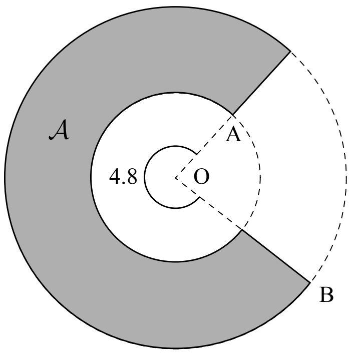
The point A lies on the circumference of the inner circle with radius r cm, where $r < 10$.

The point B lies on the circumference of the outer circle with radius 10 cm.

The reflex angle $\mathrm{A\hat{O}B}$ is 4.8 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale

$$\mathcal{A} = \tfrac{1}{2}(r_2^2 - r_1^2)\theta$$
$$= \tfrac{1}{2}(10^2 - r^2)4.8$$
$$= 2.4(100 - r^2)$$



(a) Show that the area of the “C” is given by $240 - 2.4r^2$. [2]

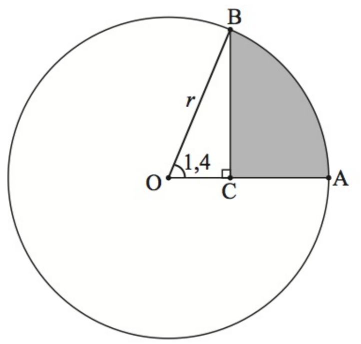
The area of the “C” is $176\,\text{cm}^2$.

(b) (i) Find the value of r. $176 = 240 - 2.4r^2 \Rightarrow r = \sqrt{\dfrac{240-176}{2.4}} = \boxed{5.16\,\text{cm}}$

(ii) Find the perimeter of the “C”. $\mathcal{P} = 2(10-r) + 4.8(10+r) = \boxed{82.45\,\text{cm}}$ [5]

Problem 2 *[8 marks]*

The following picture shows a circle of center O and radius r cm.



Points A and B lie on the circle, and $\widehat{\mathrm{AOB}} = 1.4\,$radians.

Point C is on [OA] and $\widehat{\mathrm{BOC}} = \frac{\pi}{2}\,$radians.

2.1 $\mathrm{OC} = r\cos(1.4)$, because OBC is rectangle, $\cos(1.4) = \frac{\text{adj}}{\text{hyp}}$

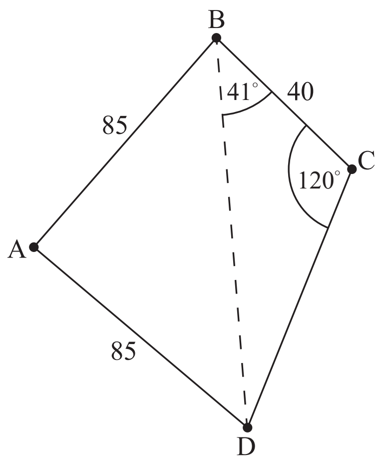
2.2 The area of the greay region is $25\text{cm}^2 = \frac{1}{2}1.4r^2 \Rightarrow r = \sqrt{\frac{25}{0.7}} = 5.98\text{cm}$

Problem 3 *[17 marks]*

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\text{AB} = 85\,\text{m},\ \text{AD} = 85\,\text{m},\ \text{BC} = 40\,\text{m},\ \text{C}\hat{\text{B}}\text{D} = 41^\circ,\ \text{B}\hat{\text{C}}\text{D} = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $\text{B}\hat{\text{A}}\text{D}$, correct to five significant figures. [3]

The size of angle $\text{B}\hat{\text{A}}\text{D}$ rounds to 77°, correct to the nearest degree. Use $\text{B}\hat{\text{A}}\text{D} = 77^\circ$ for the rest of this question.

(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(a) i) $\widehat{\text{BDC}} = 180^o - 41^o - 120^o = \boxed{19^o}$

ii) $\frac{\sin(120^o)}{\text{BD}} = \frac{\sin(19^o)}{40} \Rightarrow \text{BD} = 40\frac{\sin(120^o)}{\sin(19^o)} = \boxed{106.4m}$

(This question continues next page !!)

(b) $\widehat{\text{BAD}} = \arccos\left(\frac{\text{BD}^2 - 2\times 85^2}{-2\times 85^2}\right) = \boxed{77.5^o}$

(continuing ...)

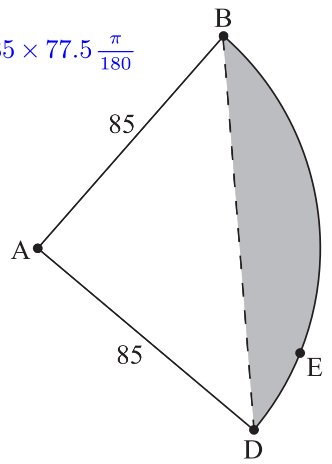
A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

d) $\mathrm{AE} = \boxed{85m}$

diagram not to scale

e) perimeter $= 2 \times 85 + 85 \times 77.5 \frac{\pi}{180}$

$= \boxed{305m}$



f)

$\mathcal{A}_{\text{sector}} = \frac{1}{2} 85^2\ 77.5 \frac{\pi}{180} = 4886 m^2$

$\mathcal{A}_{\text{ABD}} = \frac{1}{2} 85^2 \sin(77.5) = 4527 m^2$

$\mathcal{A}_{\text{DED}} = 4886 - 4527 \cong \boxed{1360 m^2}$

(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

Problem 4 *[9 marks]*

The following diagram shows a circle with centre O and radius 6 cm.

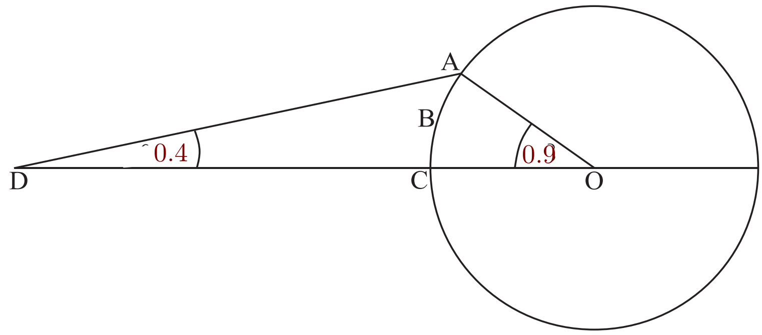


diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC). Angle ADC = **0.4** radians and angle AOC = 0.**9** radians.

(a) Find AD. $\frac{\text{AD}}{\sin(0.9)} = \frac{r}{\sin(0.4)} \Rightarrow \text{AD} = 6\frac{\sin(0.9)}{\sin(0.4)} = \boxed{12.07\text{cm}}$

(b) Find OD. $\frac{\text{OD}}{\sin(\pi - 0.9 - 0.4)} = \frac{r}{\sin(0.4)} \Rightarrow \text{OD} = 6\frac{\sin(1.842)}{\sin(0.4)} = \boxed{14.84\text{cm}}$

(c) Find the area of sector OABC. $\mathcal{A}_1 = r^2\frac{0.9}{2} = \boxed{16.2\text{cm}^2}$

(d) Find the area of region ABCD. $\mathcal{A}_2 = \frac{\text{AD} \times \text{DO}}{2}\sin(0.4) - \mathcal{A}_1 = 34.88 - 16.2 = \boxed{18.68\text{cm}^2}$

Problem 5 *[9 marks]*

The binomial expansion of $(1 + kx)^n$ is given by $1 + \frac{9x}{2} + 15k^2x^2 + \ldots + k^n x^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

$$(1+kx)^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j}(kx)^j = \binom{n}{0} + \binom{n}{1}kx + \binom{n}{2}k^2x^2 + \ldots + \binom{n}{n}k^n x^n$$

$$= 1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \ldots + k^n x^n$$

by comparison with what is given in the question : $nk = \frac{9}{2}$ and $\frac{n(n-1)}{2} = 15$

then n is solution of $n^2 - n - 30 = 0 \quad \Rightarrow \boxed{n = 6}$ and $k = \frac{9}{2n} = \boxed{\frac{3}{4}}$

Bonus [+2]

Find the exact value of θ , $\frac{\pi}{2} \leqslant \theta \leqslant \pi$ (rad.) such that $4^{\sin(\theta)} = 2$