MATHS AA SL IB$_1$

Easter Examinations

PAPER II

Friday 22 March 2024

Duration : 1h 30min

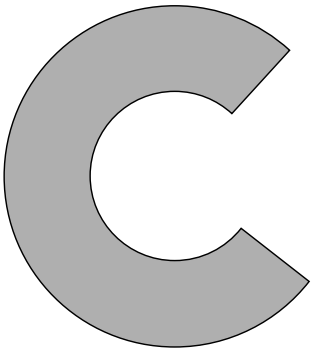
Nom/Name _____

5 questions Total : / 50 marks

The use of a calculator is not permitted for this paper

Problem 1 *[7 marks]*

A company is designing a new logo in the shape of a letter "C".



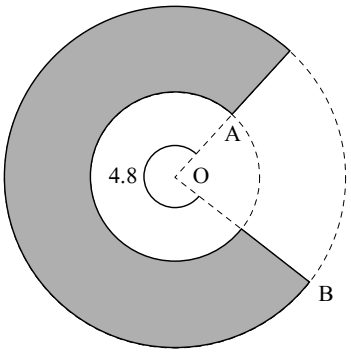
The letter "C" is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius r cm, where $r < 10$.

The point B lies on the circumference of the outer circle with radius 10 cm.

The reflex angle AÔB is 4.8 radians. The letter "C" is shown by the shaded area in the following diagram.

diagram not to scale



(a) Show that the area of the "C" is given by $240 - 2.4r^2$. [2]

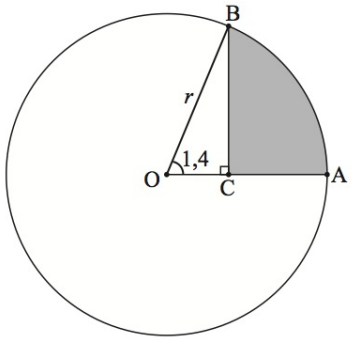
The area of the "C" is $176\,\text{cm}^2$.

(b) (i) Find the value of r.

(ii) Find the perimeter of the "C". [5]

Problem 2 *[8 marks]*

The following picture shows a circle of center O and radius r cm.



Points A and B lie on the circle, and $\widehat{\mathrm{AOB}} = 1.4$ radians.

Point C is on [OA] and $\widehat{\mathrm{BOC}} = \frac{\pi}{2}$ radians.

2.1 Show that $\mathrm{OC} = r\cos(1.4)$

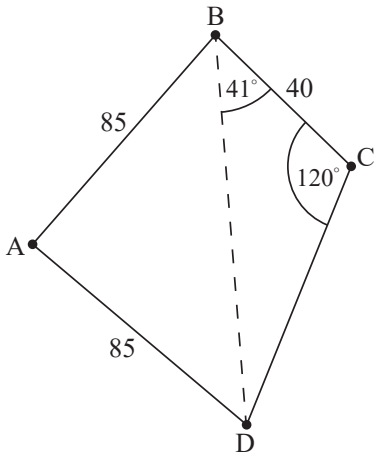
2.2 The area of the greay region is $25\mathrm{cm}^2$. Find the value of r.

Problem 3 *[17 marks]*

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB} = 85\,\mathrm{m},\ \mathrm{AD} = 85\,\mathrm{m},\ \mathrm{BC} = 40\,\mathrm{m},\ \mathrm{C\hat{B}D} = 41^\circ,\ \mathrm{B\hat{C}D} = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $\mathrm{B\hat{A}D}$, correct to five significant figures. [3]

The size of angle $\mathrm{B\hat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\hat{A}D} = 77^\circ$ for the rest of this question.

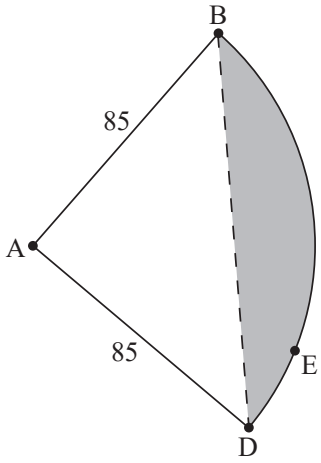
(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues next page !!)

(continuing ...)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

Problem 4 *[9 marks]*

The following diagram shows a circle with centre O and radius 6 cm.

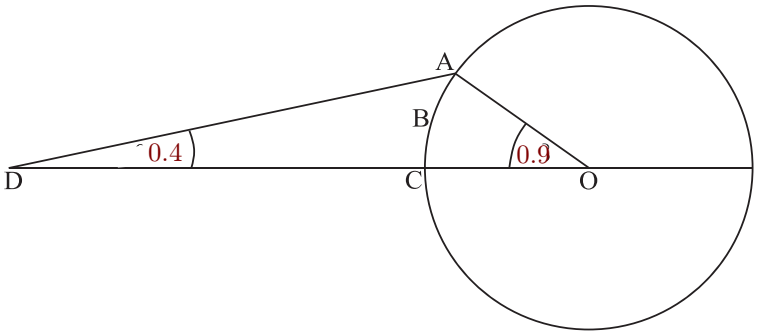


diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).
Angle ADC = 0.4 radians and angle AOC = 0.9 radians.

(a) Find AD.

(b) Find OD.

(c) Find the area of sector OABC.

(d) Find the area of region ABCD.

Problem 5 *[9 marks]*

The binomial expansion of $(1+kx)^n$ is given by $1+\frac{9x}{2}+15k^2x^2+\ldots+k^nx^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

Bonus [+2]

Find the exact value of θ, $\frac{\pi}{2} \leqslant \theta \leqslant \pi$ (rad.) such that $4^{\sin(\theta)} = 2$