Christmas Examination

Wednesday 11 Dec.2024

Maths SL IB$_2$

Part 1

(7 Problems 93 marks)

Name : __________

A calculator is not allowed for this first part

Problem 1 [/ 9 marks]

We consider the function $f(x) = 2\cos(x)e^x$

a) Find the derivative of $f(x)$

b) What is the gradient of the tangent to the curve $y = f(x)$, at $x = 0$?

c) The graph of f has an *horizontal* tangent line at x_0 ,with $0 < x_0 < \frac{\pi}{2}$

Find x_0.

Problem 2 [/ 12 marks]

Let us consider

$f(x) = (x-1)^2 e^{2x}$

1) Find the values of x such that the tangent to the curve of equation $y = f(x)$ is *horizontal.*

2) Show that $f''(x) = 2e^{2x}(2x^2 - 1)$

3) Using your previous result, or otherwise, determine for each of your solutions to (1) whether it is a *maximum* or a *maximum*.

Problem 3 [/ 16 marks]

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$.

(a) (i) Write down $f'(2)$.

(ii) Find $f(2)$. [4]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

(b) Show that the graph of g has a gradient of 6 at P. [5]

(c) Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. Find the y-coordinate of Q. [7]

Problem 4 [/ 14 marks]

A tennis ball is dropped from an hight tower.

It velocity is given by

$v(t) = 72(1 - e^{-0.16t})$ where v is in m/s and t in seconds

a) Find the velocity of the ball at [2]

i) $t = 0$

ii) $t = 8$ sec

b) [6]

i) Find an expression for the acceleration a as a function of t.

ii) What is the value of a when $t = 0$?

c) [6]

i) As t becomes *large*, what values does v approach ?

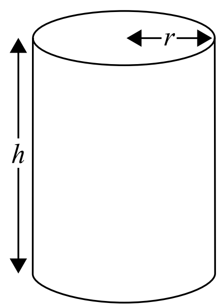
ii) As t becomes *large*, what values does a approach ?

iii) Explain the relationship between the answers to part (i) and part (iii)

Problem 5 [/ 15 marks]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20π cm^3.

diagram not to scale



(a) Express h in terms of r. [2]

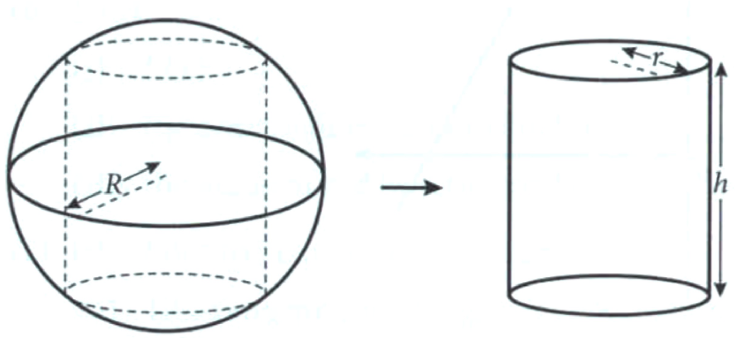
The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is C.

(b) Show that $C = 20\pi r^2 + \dfrac{320\pi}{r}$. [4]

(c) Given that there is a minimum value for C, find this minimum value in terms of π. [9]

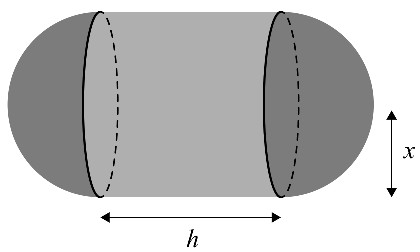
Problem 6 [/ 13 marks]

Find the hight h and the base radius r of the largest right circular cylinder that can be made by cutting it away from a sphere with a radius of R.



Problem 7 [/ 14 marks]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height h cm and radius x cm. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is $41\,\text{cm}^3$.

(a) Show that the total surface area, $S\,\text{cm}^2$, of the solid is given by $S = \dfrac{82}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\dfrac{\mathrm{d}S}{\mathrm{d}x}$.

(ii) Hence, find the **exact** value of a. [5]

(c) (i) Find an expression for $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$.

(ii) Use the second derivative of S to justify that S is a minimum when $x = a$.

(iii) Find the minimum surface area of the solid. [6]