**Christmas Examination**

Wednesay 13 December 2023

Maths SL $IB_2$

**Part 2**

( 7 Problems 74 marks )

Name: ____________

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## Problem 1

[ / 15 marks ]

The function $f$ is defined by $f(x) = \cos^2 x - 3\sin^2 x,\ 0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = f(x)$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [3]

## Problem 2

[ / 13 marks ]

Consider the function $f$ defined by $f(x) = 90\,\mathrm{e}^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of $f$ and the line $y = x$ intersect at point P.

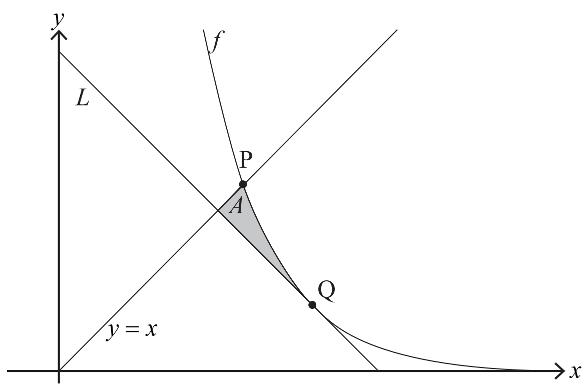
(a) Find the $x$-coordinate of P. [2]

The line $L$ has a gradient of $-1$ and is a tangent to the graph of $f$ at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of $L$ is $y = -x + 2\ln 45 + 2$. [2]

The shaded region $A$ is enclosed by the graph of $f$ and the lines $y = x$ and $L$.




(d) (i) Find the $x$-coordinate of the point where $L$ intersects the line $y = x$.

(ii) Hence, find the area of $A$. [5]

**Problem 3** [ / 15 marks ]

Consider the function $h(x)=\sqrt{4x-2}$, for $x\geq\frac{1}{2}$.

(a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.

(ii) Write down the range of $h^{-1}(x)$. [5]

(b) The graph of $h$ intersects the graph of $h^{-1}$ at two points.

Find the $x$-coordinates of these two points. [3]

(c) Find the area enclosed by the graph of $h$ and the graph of $h^{-1}$. [2]

(d) Find $h'(x)$. [2]

(e) Find the value of $x$ for which the graph of $h$ and the graph of $h^{-1}$ have the same gradient. [3]

**Problem 4** [ / 7 marks ]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, at time $t$ seconds is given by $v=\frac{(t^2+1)\cos t}{4}$, $0\leq t\leq 3$.

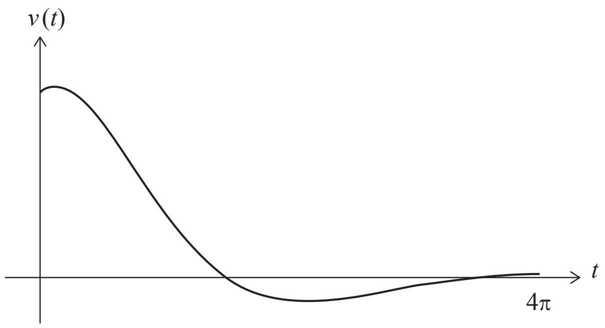
(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\mathrm{m\,s^{-2}}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

**Problem 5** [ / 6 marks ]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, at time $t$ seconds is given by $v(t)=4\mathrm{e}^{-\frac{t}{3}}\cos\left(\frac{t}{2}-\frac{\pi}{4}\right)$, for $0\leq t\leq 4\pi$. The graph of $v$ is shown in the following diagram.




Let $t_1$ be the first time when the particle's **acceleration** is zero.

(a) Find the value of $t_1$. [2]

Let $t_2$ be the **second** time when the particle is instantaneously at rest.

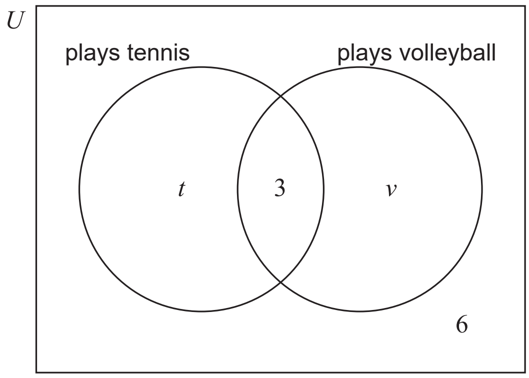
(b) Find the value of $t_2$. [2]

(c) Find the distance travelled by the particle between $t=t_1$ and $t=t_2$. [2]

**Problem 6** [ / 6 marks ]

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events "plays tennis" and "plays volleyball".
The values $t$ and $v$ represent numbers of students.




(a) (i) Find the value of $t$.

(ii) Find the value of $v$. [4]

(b) Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both. [2]

**Problem 7** [ / 12 marks ]

**(a)** A bag contains two gold balls and one silver balls.

<u>Two</u> balls are drawn at random from the bag, with replacement.

( that means we replace each ball in the bag after lokking at its colour )

Let $X$ be the number of gold balls drawn from the bag.

1. Find P($X$=0). [1]
2. Find P($X$=1). [1]
3. Find P($X$=2). [1]
4. Find P(X =1 assuming the first ball is gold ). [3]

**(b)** Now we consider that <u>four</u> balls are drawn from the same bag, still with replacement.

1. Find the probability P( first ball is gold, second is silver, thirs is gold and fourth is gold ).
2. Find the probability that exactly 3 of the 4 balls are gold.