Christmas Examination

Wednesay 14 Dec. 2022
Duration : 90 min

Maths SL IB$_2$

Part 1

(7 Problems 74 marks)

Name: ___________

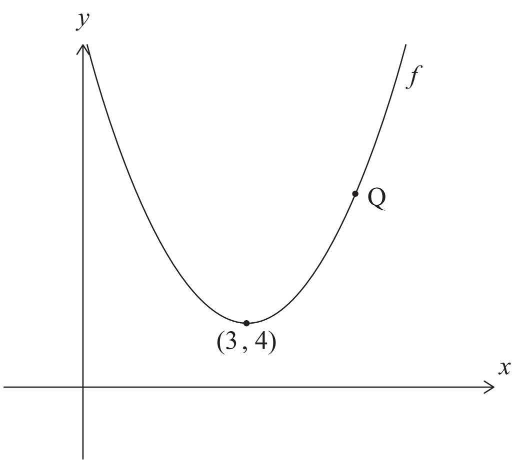
A calculator is not allowed for this fisrt part

Problem 1

[/ 15 marks]

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3, 4)$, and it passes through point Q as shown.



(a) Write down the equation of the axis of symmetry. [1]

(b) The function can be written in the form $f(x) = a(x - h)^2 + k$.

 (i) Write down the values of h and k.

 (ii) Point Q has coordinates $(5, 12)$. Find the value of a. [4]

The line L is tangent to the graph of f at Q.

(c) Find the equation of L. [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

(d) Find the values of d for which g is an increasing function. [3]

(e) Find the values of x for which the graph of g is concave-up. [3]

Problem 2

[/ 6 marks]

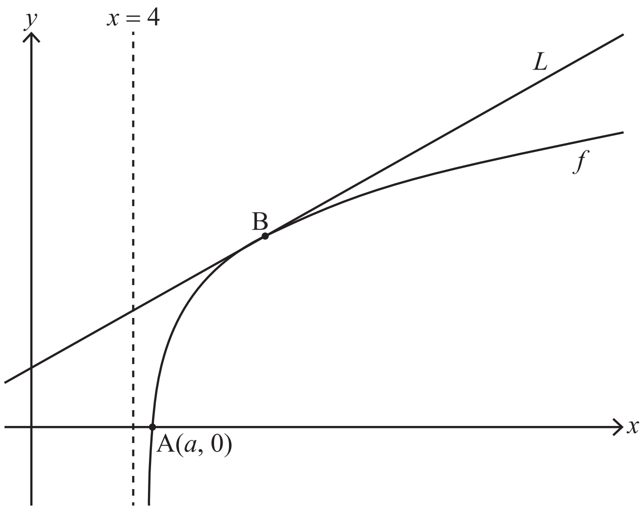
Given that $\dfrac{dy}{dx} = \cos\left(x - \dfrac{\pi}{4}\right)$ and $y = 2$ when $x = \dfrac{3\pi}{4}$, find y in terms of x.

Problem 3

[/ 9 marks]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



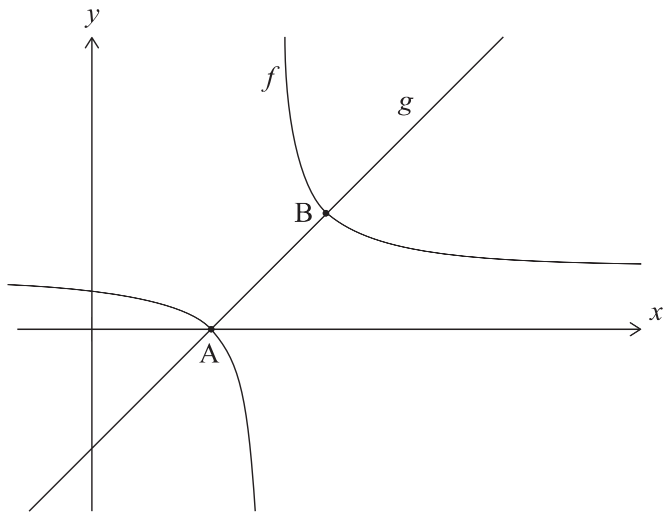
(a) Find the exact value of a. [3]

(b) Given that the gradient of L is $\dfrac{1}{3}$, find the x-coordinate of B. [6]

Problem 4 [/ 15 marks]

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

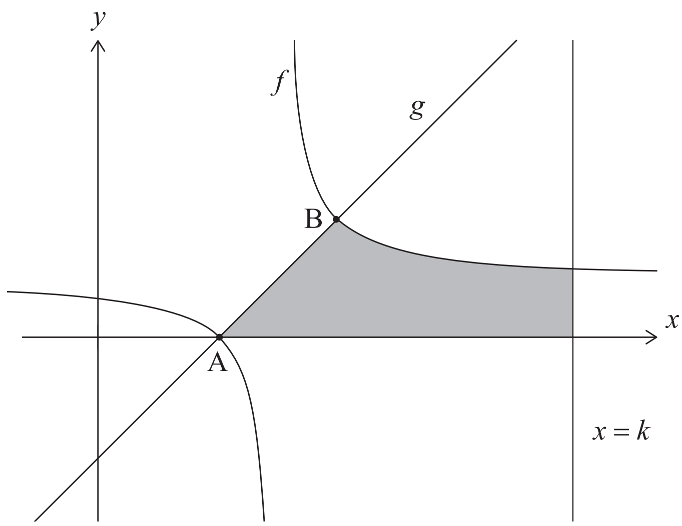
The following diagram shows the graphs of f and g.



The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

(a) Find the coordinates of B. [5]

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

(b) Find the value of k and the value of p. [10]

Problem 5 [/ 16 marks]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\dfrac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

Problem 6 [/ 5 marks]

Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

Problem 7 [/ 8 marks]

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

(a) Find the probability, in terms of n, that the game will end on her

(i) first draw;

(ii) second draw. [4]

(b) Let $n = 5$. Find the probability that the game will end on her

(i) third draw;

(ii) fourth draw. [4]