MATHS AA

June Exam

Total : / 138 marks

Friday 14 June 2024

Total duration : 3 hours

Nom/Name _______________

Problem 1

[/5 marks]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

Problem 2

[/9 marks]

An *arithmetic* sequence has $u_1 = \log_c(r)$ and $u_2 = \log_c(r^2 s)$

(a) Show that the *constant difference* is $d = \log_c(r) + \log_c(s)$ [2]

(c) Let $r = c$ and $s = c^7$

Find the value of $\sum_{n=6}^{10} u_n$ [7]

Problem 3

[/7 marks]

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ldots + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

(b) Find the possible values of x. [5]

Problem 4

[/6 marks]

Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$.

Problem 5

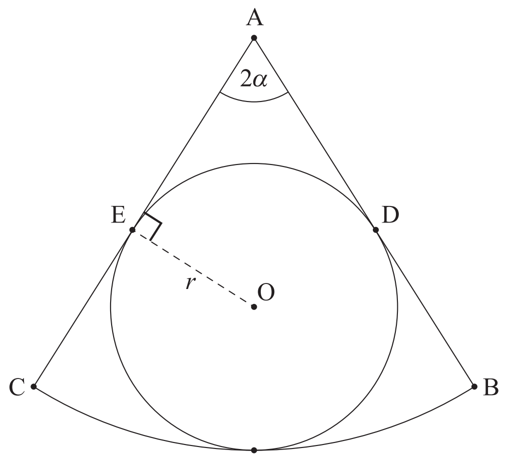
[/9 marks]

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

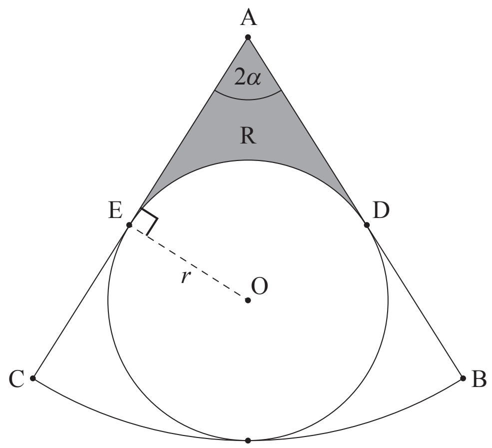
A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\dfrac{r^2}{\tan\alpha}$. [4]



(b) (i) Find $\mathrm{D\hat{O}E}$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

Problem 6 *[/14 marks]*

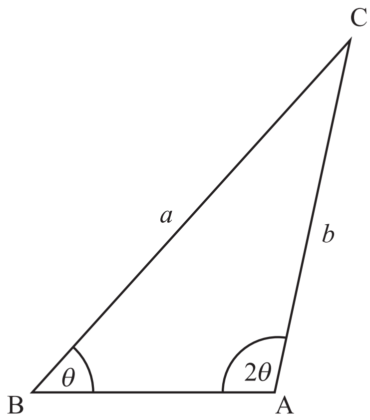
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(i) $\sin\theta$;

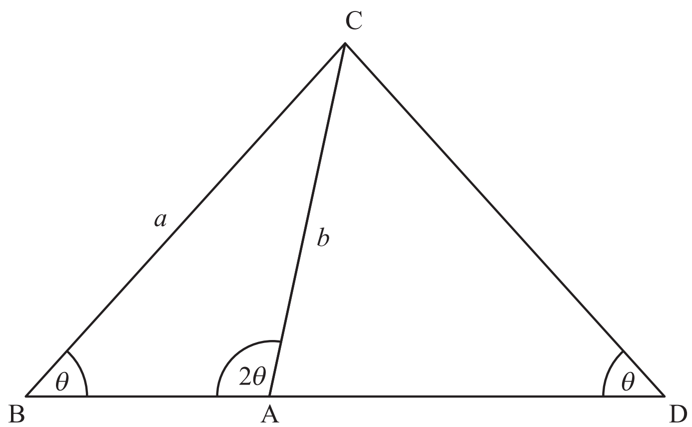
(ii) $\sin 2\theta$. [4]

The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

[BA] is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin C\hat{A}D$. [3]

(d) Find the area of triangle DAC, in terms of a. [5]

Problem 7 *[/13 marks]*

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

(a) All the values of $f(x)$ are between k and m. Find the vales of k and m. [3]

Let $g(x) = 5f(2x)$.

(b) All the values of $g(x)$ are between p and q. Find the vales of p and q. [2]

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

(c) (i) Find the value of b and of c.

(ii) Find the period of g. [5]

(d) The equation $g(x) = 12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions. [3]

Problem 8 *[/8 marks]*

(a) Assuming $\log_9(x) = \log_3(y)$, write y in terms of x.

Help : You could use the identity $\log_a(x) = \frac{\log_b(x)}{\log_b(x)}$ to transform the left member of the equation. [2]

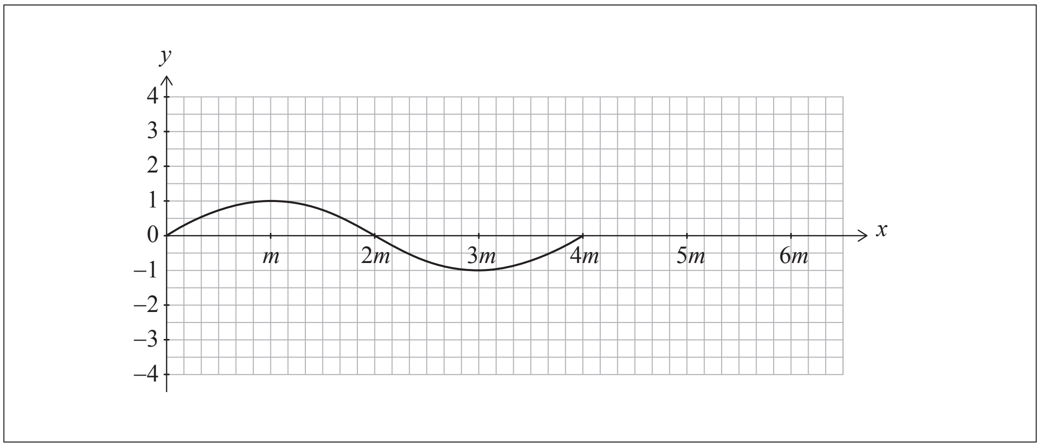
(b) Show that $\log_9(\cos(2x)+2) = \log_3\left(\sqrt{\cos(2x)+2}\right)$ [2]

(c) Hence of otherwise solve $\log_3(2\sin(x)) = \log_9(\cos(2x)+2)$ for $0 < x < \frac{\pi}{2}$ [4]

Problem 9

[/6 marks]

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0$, $2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

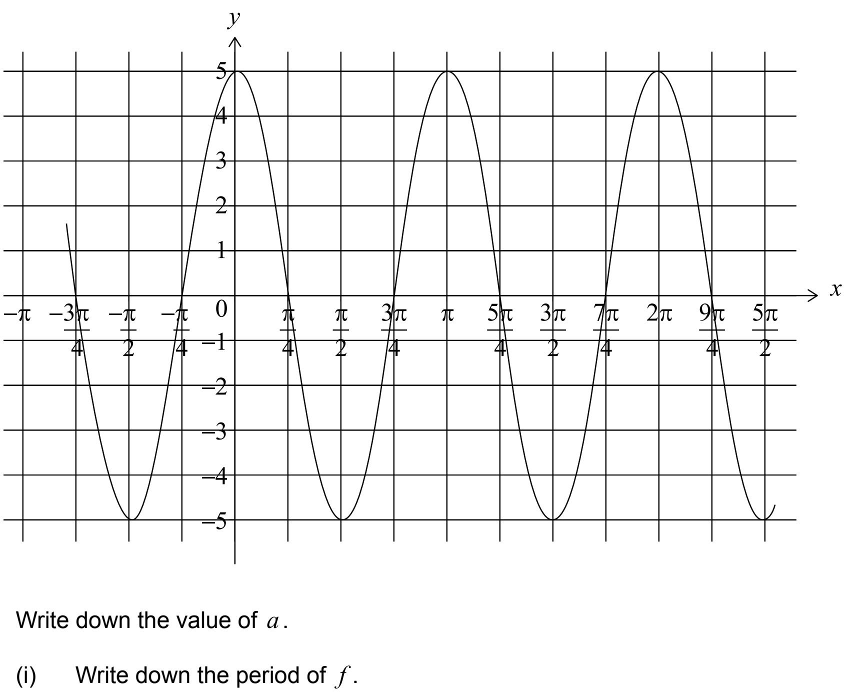
The function g is defined by $g(x) = 3\sin\frac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

Problem 10

[/7 marks]

Consider the function $f(x) = a\cos(bx)$, with a, $b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

(b) (i) Write down the period of f.

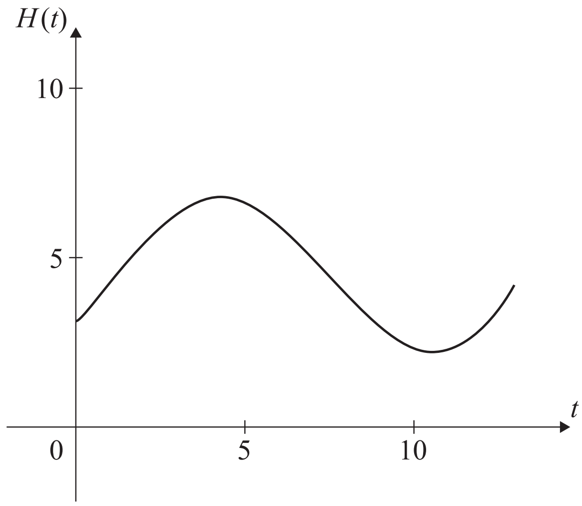
(ii) Hence, find the value of b. [3]

(c) Find the value of $f\left(\frac{\pi}{6}\right)$. [3]

Problem 11 *[/13 marks]*

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin\left(b(t-c)\right) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

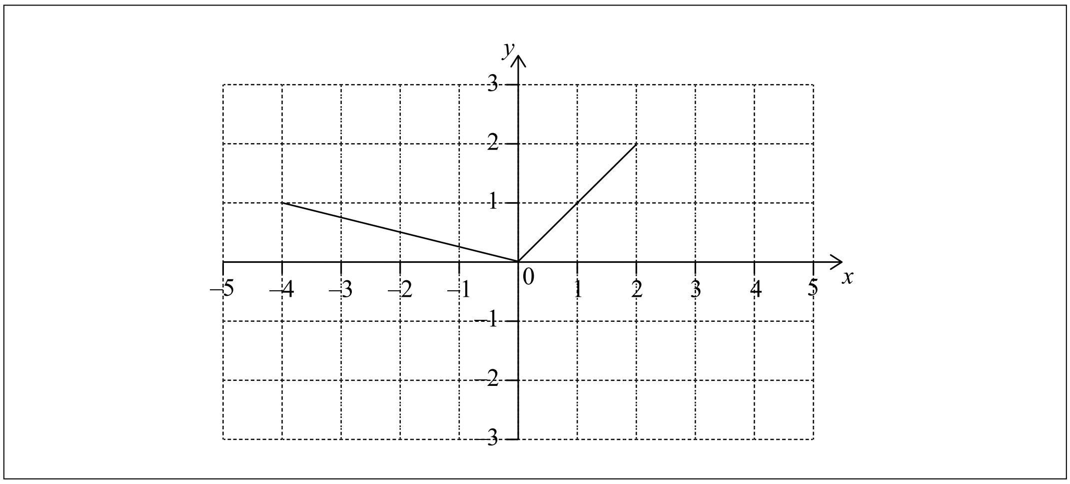
(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

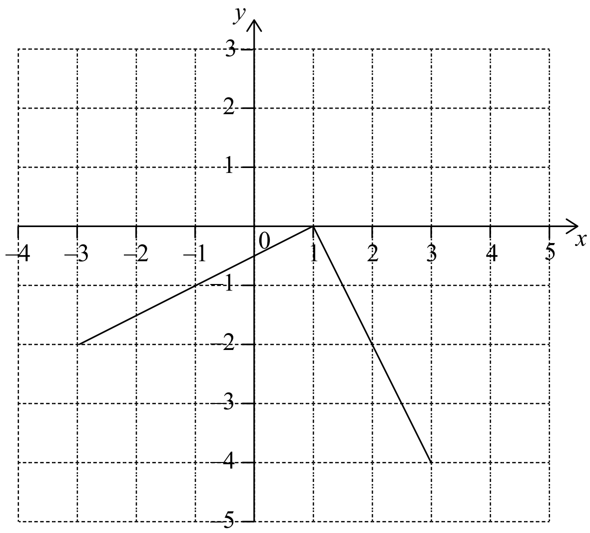
Problem 12 *[/6 marks]*

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, g, can be written in the form $g(x) = a \times f(x+b)$. The following diagram shows the graph of g.



Write down the value of a and of b. [4]

Problem 13 *[/11 marks]*

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

(a) Find the value of b. [3]

(b) The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]

(c) The graph of g is transformed by the two following consecutives transformations to obtain the graph of h : [4]

i) a horizontal stretch of scale factor 2

ii) a reflexion by the y-axis.

Write down the function $h(x)$.

Problem 14 *[/6 marks]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

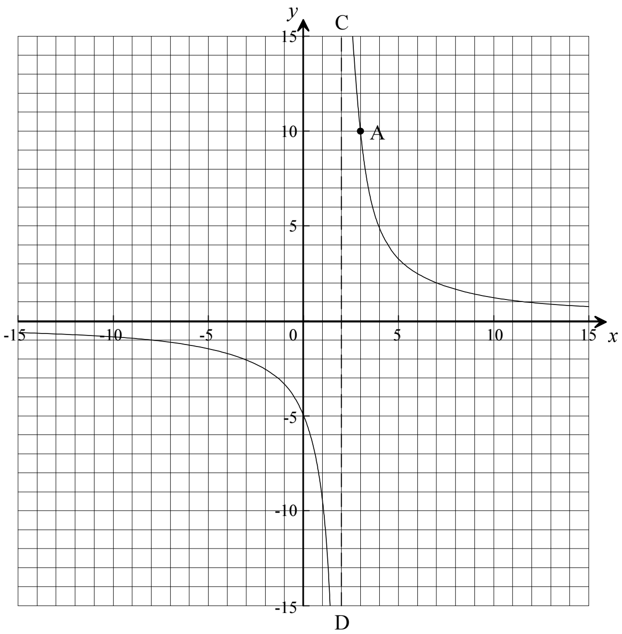
(iii) q. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point $A(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

Problem 15

[/7 marks]

(a) The diagram shows part of the graph of the function $f(x) = \dfrac{q}{x-p}$. The curve passes through the point $\mathrm{A}(3, 10)$. The line (CD) is an asymptote.

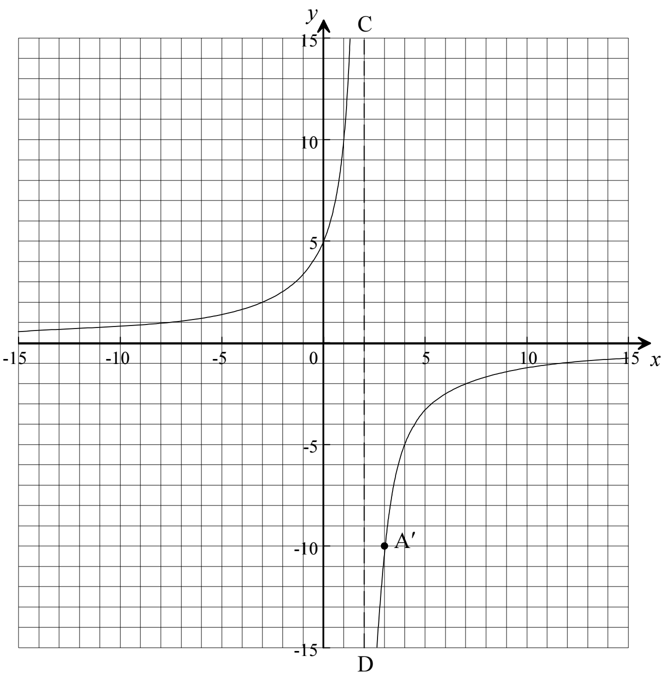


Find the value of

(i) p;

(ii) q.

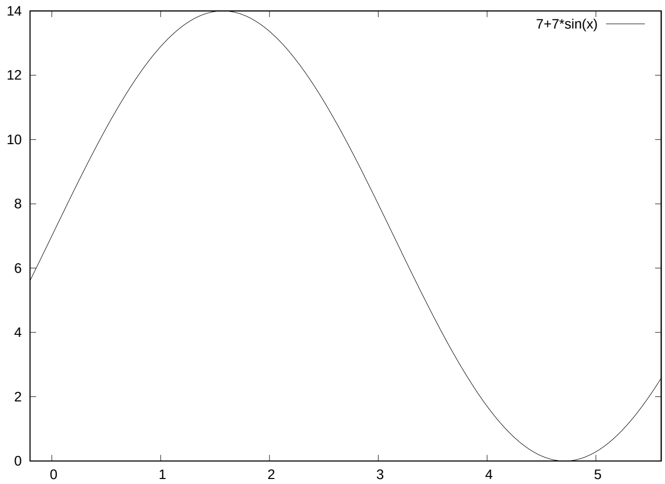
(b) The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to $\mathrm{A}'(3, -10)$.



Give a full geometric description of the transformation.

Problem 16 *[/11 marks]*

Let $f(x) = 7 + 7\sin x$. Part of the graph of f is shown below.



(a) What is the maximal value for $f(x)$? [1]

(b) Solve for $0 \leq x < 2\pi$ [5]

(i) $f(x) = 7$

(ii) $f(x) = 0$

(c) Write down the exact value of the $x-$intercept of f , for $0 \leq x < 2\pi$. [2]

Let $g(x) = 7 + 7\sin\left(x - \frac{\pi}{2}\right)$.

The graph of f is transformed to the graph of g.

(d) Give a full geometric description of this transformation [3]