



## Christmas Examination

Maths AA SL IB<sub>1</sub> Part 1  
( 7 Problems )

Tot: / 40



Tuesday 12 December 2023

Name : \_\_\_\_\_

You are not permitted access to any calculator for this paper.

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### Problem 1

[ /6 marks]

Solve the following equations:

i)  $\frac{x(17-2x)-1}{5} = 4$  [2]

ii)  $\frac{\log_3(x)(17-\log_3(x^2))-1}{5} = \log_2(16)$  [4]

### Problem 2

[ /6 marks]

(a) Calculate the value of each of the following logarithms:

(i)  $\log_2 \frac{1}{16}$ ;

(ii)  $\log_9 3$ ;

(iii)  $\log_{\sqrt{3}} 81$ .

(b) It is given that  $\log_{ab} a = 3$ , where  $a, b \in \mathbb{R}^+$ ,  $ab \neq 1$ .

(i) Show that  $\log_{ab} b = -2$ .

(ii) Hence find the value of  $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$ .

### Problem 3

[ /4 marks]

Let us consider  $a = \log_3(2)$  and  $b = \log_3(5)$ ,  $x = \log_3(1 + \frac{3}{125})$  and  $y = \log_3(100)$ .

Find an expression for  $x$  and an expression for  $y$ , in terms of  $a$  and  $b$ .

**Problem 4**

[ /5 marks]

The  $n^{\text{th}}$  term of an *arithmetic* sequence is given by  $u_n = 15 - 3n$ .

- (a) State the value of the first  $u_1$ .
- (b) Given that the  $n^{\text{th}}$  term of this sequence is  $-33$ , find the value of  $n$ .
- (c) Find the *common difference*,  $d$ .

**Problem 5**

[ /6 marks]

An *geometric* sequence has first term  $u_1 = a$  and second term  $u_2 = 1a^2 - 3a$ , where  $a > 0$ .

- (a) Find the *constant ratio*  $r$  in terms of  $a$ .

Let us consider the series  $s_n = \sum_{k=1}^n u_k$ .

- (b) Give the *general term* for  $s_n$ .
- (c) Find the values of  $a$  for which the sum to infinity exists.

Hint : There is a formula about this condition in the IB booklet !

**Problem 6**

[ /7 marks]

The expansion of  $(x + h)^8$ , where  $h > 0$ , can be written as  $x^8 + ax^7 + bx^6 + cx^5 + dx^4 + \dots + h^8$ , where  $a, b, c, d, \dots \in \mathbb{R}$ .

- (a) Find an expression, in terms of  $h$ , for
  - (i)  $a$ ;
  - (ii)  $b$ ;
  - (iii)  $d$ . [4]
- (b) Given that  $a, b$ , and  $d$  are the first three terms of a geometric sequence, find the value of  $h$ . [3]

**Problem 7**

[ /6 marks]

Consider the binomial expansion  $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \dots + 1$

where  $x \neq 0, a \neq 0, b \neq 0, a, b \in \mathbb{N}$

- (a) Show that  $b=21$
- (b) The third term in the expansion is the *mean* of the second term and the fourth term in the expansion. Find the possible values of  $x$ .