Christmas Examination

Maths AA SL IB$_1$ Part 2

(7 Problems)

Tot: / 40

Tuesday 13 Dec. 2022

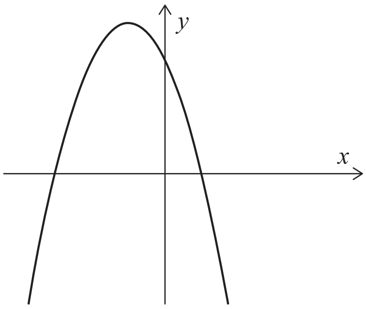
** ANSWERS **

A graphic display calculator may be required for this paper

Problem 1

[7marks]

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



(a) For the graph of f

(i) find the x-coordinates of the x-intercepts; $\boxed{\text{S}=\{-3, 1\}}$

(ii) find the coordinates of the vertex. $\boxed{\text{V}:(-1, 8)}$ [5]

The function f can be written in the form $f(x) = -2(x-h)^2 + k$.

(b) Write down the value of h and the value of k. $\boxed{h=-1 \quad \text{and} \quad k=8}$ [2]

Problem 2

[4marks]

$$\left(3x^2 - \frac{k}{x}\right)^9 = -\frac{k^9}{x^9} + \frac{27k^8}{x^6} - \frac{324k^7}{x^3} + 2268k^6 - 10206k^5x^3 + 30618k^4x^6 - 61236k^3x^9 + \ldots.$$

If we want the coefficient of the term in x^6 to be 6048 then $k^4 = \frac{6048}{30618}$ then $\boxed{k = 2/3}$

Problem 3

[9marks]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. $u_1 = S_2 = \frac{2}{3}\left(\frac{7}{8}\right) = \boxed{\frac{7}{12}}$ [2]

(b) Find S_∞. $S_n = u_1 \frac{\left(\frac{7}{8}\right)^n - 1}{\left(\frac{7}{8}\right) - 1} \Rightarrow S_\infty = \frac{7}{12} \frac{0-1}{\left(\frac{7}{8}\right) - 1} = \frac{7}{12} \cdot \frac{1}{\frac{1}{8}} = \boxed{\frac{14}{3}}$ [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. $\Leftrightarrow S_n = \frac{56}{13} - \frac{1}{1000}$ [4]

$$\Leftrightarrow \frac{7}{12} \frac{\left(\frac{7}{8}\right)^n - 1}{\left(\frac{7}{8}\right) - 1} = \frac{13997}{3000} \quad \Leftrightarrow \frac{\left(\frac{7}{8}\right)^n - 1}{\left(\frac{7}{8}\right) - 1} = \frac{13997}{1750} \Leftrightarrow \left(\frac{7}{8}\right)^n = 1 - \frac{13997}{14000} = \frac{3}{14000} \Leftrightarrow n = \log_{\frac{7}{8}}\left(\frac{3}{14000}\right) = 63.2675$$

as n is an integer, $\boxed{n = 64}$

Problem 4

[5marks]

$u_n = 60 - 2.5(n-1) = 62.5 - 2.5n$

(a) $u_k = 0 \Leftrightarrow 62.5 - 2.5k = 0 \Leftrightarrow k = \frac{62.5}{2.5} = \boxed{25}$

(b) $S_n = \frac{n}{2}(2 \times 60 + (n-1) \times (-2.5)) = n\left(60 - (n-1) \times \frac{5}{4}\right) = -\frac{5}{4}n^2 + \left(60 + \frac{5}{4}\right)n = n\left(\frac{245}{4} - \frac{5}{4}n\right)$

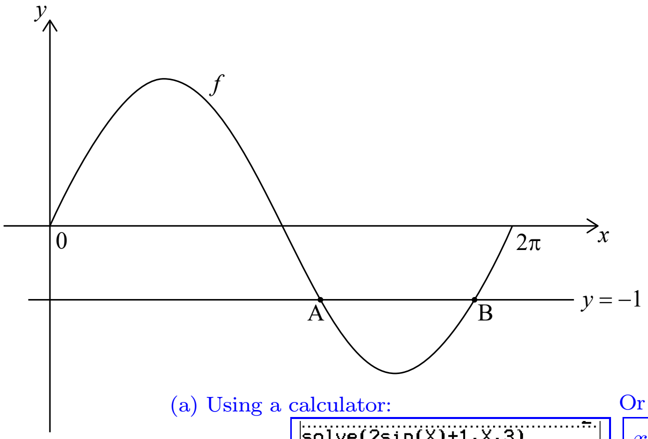
then $S_n = 0$ for $n = 0$ or $n = 49$

then *maximal* value for n close to $\frac{49}{2}$, that is : $\boxed{S_{24} = S_{25} = 750}$

Problem 5

[7marks]

Consider the graph of the function $f(x) = 2\sin x$, $0 \le x < 2\pi$. The graph of f intersects the line $y = -1$ exactly twice, at point A and point B. This is shown in the following diagram.



(a) Using a calculator:

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solve(2sin(X)+1,X,3)
                3.665191429
solve(2sin(X)+1,X,6)
                5.759586532
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Or by Sarina's method $\boxed{x_A = \frac{7\pi}{6}, x_B = \frac{11\pi}{6}}$

(a) Find the x-coordinate of A and of B. [4]

Consider the graph of $g(x) = 2\sin px$, $0 \le x < 2\pi$, where $p > 0$.

(b) $\boxed{p < \frac{5}{12}}$

(b) Find the greatest value of p such that the graph of g does not intersect the line $y = -1$. [3]

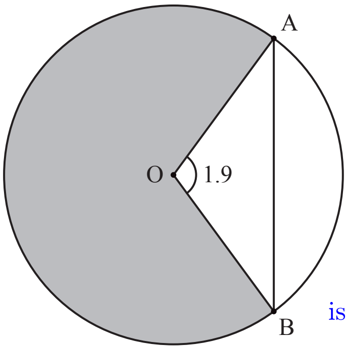
Problem 6

[4marks]

Points A and B lie on the circle and $\text{A}\ddot{\text{O}}\text{B} = 1.9$ radians.

The radius is $r = 1$.

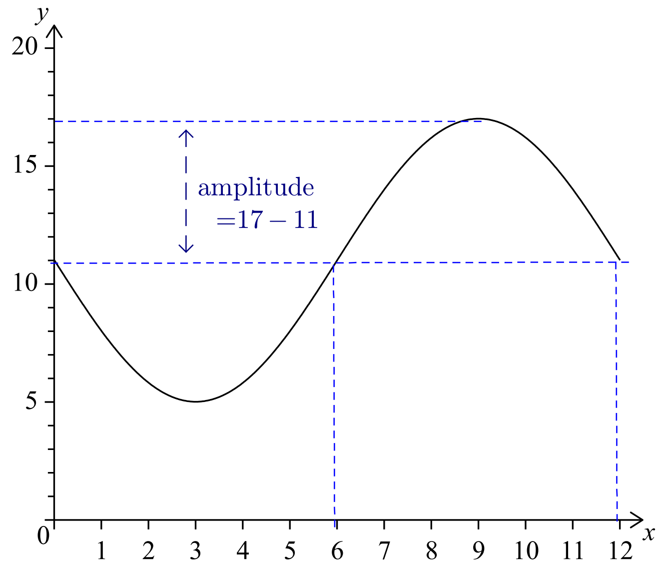
diagram not to scale



The distance AB is $d_{\text{AB}} = 2\sin\left(\frac{1.9}{2}\right) = \boxed{1.63}$

Problem 7 [6marks]

The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \le x \le 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(a) (i) Find the value of c. $\boxed{c=11}$

(ii) Show that $b = \frac{\pi}{6}$. The period is $T=12 = \frac{2\pi}{b} \Rightarrow \boxed{b = \frac{\pi}{6}}$

(iii) Find the value of a. $\boxed{a=6}$ [6]