



## Christmas Examination

Maths AA SL IB<sub>1</sub> Part 2  
( 7 Problems )

Tot: / 40



Tuesday 13 Dec. 2022

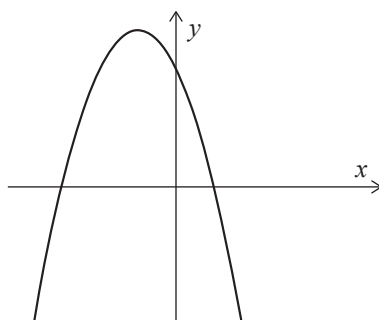
Name : \_\_\_\_\_

*A graphic display calculator may be required for this paper*

### Problem 1

[ 7marks ]

Consider the function  $f(x) = -2(x - 1)(x + 3)$ , for  $x \in \mathbb{R}$ . The following diagram shows part of the graph of  $f$ .



(a) For the graph of  $f$

- (i) find the  $x$ -coordinates of the  $x$ -intercepts;
- (ii) find the coordinates of the vertex.

[5]

The function  $f$  can be written in the form  $f(x) = -2(x - h)^2 + k$ .

(b) Write down the value of  $h$  and the value of  $k$ .

[2]

### Problem 2

[ 4marks ]

Consider the expansion of  $\left(3x^2 - \frac{k}{x}\right)^9$ , where  $k > 0$ .

The coefficient of the term in  $x^6$  is 6048. Find the value of  $k$ .

### Problem 3

[ 9marks ]

The sum of the first  $n$  terms of a geometric sequence is given by  $S_n = \sum_{r=1}^n \frac{2}{3} \left(\frac{7}{8}\right)^r$ .

(a) Find the first term of the sequence,  $u_1$ .

[2]

(b) Find  $S_\infty$ .

[3]

(c) Find the least value of  $n$  such that  $S_\infty - S_n < 0.001$ .

[4]

**Problem 4**

[ 5marks ]

An arithmetic sequence has first term 60 and common difference  $-2.5$ .

(a) Given that the  $k$ th term of the sequence is zero, find the value of  $k$ .

[2]

Let  $S_n$  denote the sum of the first  $n$  terms of the sequence.

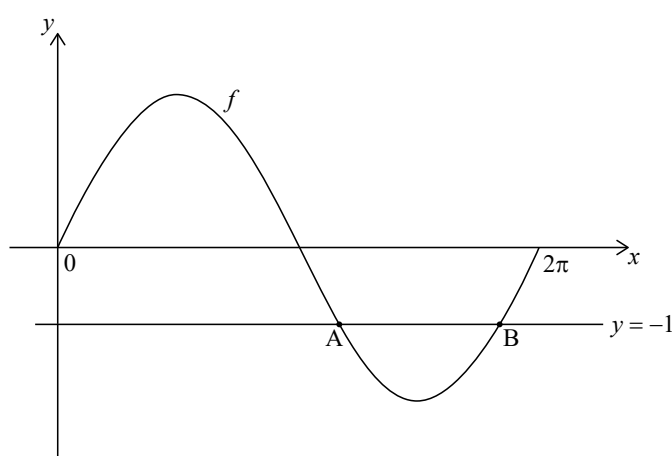
(b) Find the maximum value of  $S_n$ .

[3]

**Problem 5**

[ 7marks ]

Consider the graph of the function  $f(x) = 2 \sin x$ ,  $0 \leq x < 2\pi$ . The graph of  $f$  intersects the line  $y = -1$  exactly twice, at point A and point B. This is shown in the following diagram.



(a) Find the  $x$ -coordinate of A and of B.

[4]

Consider the graph of  $g(x) = 2 \sin px$ ,  $0 \leq x < 2\pi$ , where  $p > 0$ .

(b) Find the greatest value of  $p$  such that the graph of  $g$  does not intersect the line  $y = -1$ .

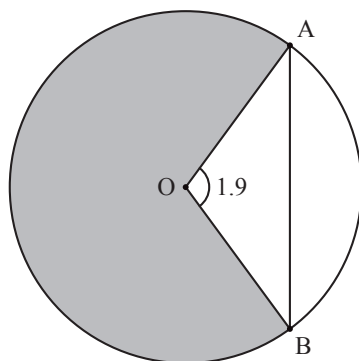
[3]

**Problem 6**

[ 4marks ]

Points A and B lie on the circle and  $\text{AOB} = 1.9$  radians.

diagram not to scale

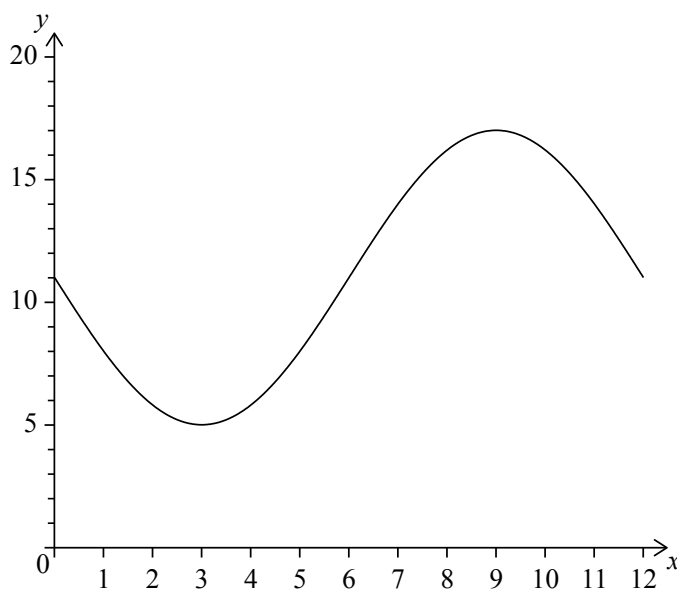


The radius is  $r = 1$ . Calculate the distance AB.

**Problem 7**

[ 6marks ]

The following diagram shows the graph of  $f(x) = a \sin bx + c$ , for  $0 \leq x \leq 12$ .



The graph of  $f$  has a minimum point at  $(3, 5)$  and a maximum point at  $(9, 17)$ .

(a) (i) Find the value of  $c$ .

(ii) Show that  $b = \frac{\pi}{6}$ .

(iii) Find the value of  $a$ .

[6]