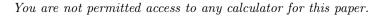
****	Christmas Examination	R
	$\begin{array}{c} \text{Maths AA SL IB}_1 \text{ Part 1} \\ \text{(7 Problems)} \end{array}$	Tuesday 13 Dec. 2022
	Tot: / 42	* ANSWERS *



Problem 1 / 5marks The nth term of an *arithmetic* sequence is given by $u_n = 15 - 3n$. (a) The value of the first term is $u_1=12$ [1](b) $u_n = -33 \Rightarrow 15 - 3n = -33 \Rightarrow$ the value of n is n = 16[2] (c) The common difference, d is -3[2]

Problem 2

- (a) The sum of these three integers is 3n, therefore for any integer it is divisible by 3.
- (b) The sum of the squares of these three integers is $(n-1)^2 + n^2 + (n+1)^3 = 3n^2 + 2$ therefore for any integer it *cannot* be divisible by 3.

Problem 3

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \dots + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$

(a)
$$(x+1)^7 = \sum_{k=0}^7 \binom{7}{k} x^k \cdot 1^{7-k} = \sum_{k=0}^7 \binom{7}{k} x^k$$
. We get the value of *b* taking $k=5$: $b = \binom{7}{5} = \boxed{21}$

(b) The third term in the expansion is the $mean^{\star}$ of the second term and the fourth term in the

In other term :
$$\binom{7}{5}x^5 = \frac{\binom{7}{6}x^6 + \binom{7}{4}x^4}{2}$$

 $\Rightarrow 21x^5 = \frac{7x^6 + 35x^4}{2} \Rightarrow 42x^5 - 7x^6 - 35x^4 = 0 \Rightarrow 6x^5 - x^6 - 5x^4 = 0$
 $\Rightarrow x^4(6x - x^2 - 5) = 0 \Rightarrow x^4(x - 1)(x - 5) = 0 \Rightarrow \boxed{x \in \{0, 1, 5\}}$

Problem 4

(a)
$$2\cos^2(x) + 5\sin(x) = 4 \Rightarrow 2(1 - \sin^2(x)) + 5\sin(x) - 4 = 0 \Rightarrow 2\sin^2(x) - 5\sin(x) + 2 = 0$$

(b) Hence
$$2\cos^2(x) + 5\sin(x) = 4 \Rightarrow 2s^2 - 5s + 2 = 0$$
. with $s = \sin(x)$
 $s = \frac{5 \pm 3}{4} = 2$ or $\frac{1}{2}$ then $\sin(x) = \frac{1}{2} \Rightarrow \boxed{x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}}$ for $0 \le x \le 3\pi$.

/ 7marks

/ 5marks

/ 6marks

Problem 5

/ 6marks

Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ Hint : You are supposed to know that $\cos(\alpha) = \frac{1}{\sqrt{2}}$ for $\alpha = \frac{\pi}{4} + 2k\pi$ or $\alpha = \frac{7\pi}{4} + 2k\pi$ Then $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi$ or $\frac{x}{2} + \frac{\pi}{3} = \frac{7\pi}{4} + 2k\pi$ $\Rightarrow \frac{x}{2} = -\frac{\pi}{12} + 2k\pi$ or $\frac{x}{2} = \frac{17\pi}{12} + 2k\pi$ $\Rightarrow x = -\frac{\pi}{6} + k\pi = \boxed{\frac{11\pi}{6} + k\pi}$ or $x = \boxed{\frac{17\pi}{6} + k\pi}$ you can also write $\boxed{S = \{\frac{11\pi}{6} + k\pi\}_{k \in \mathbb{Z}}}$

Problem 6

/ 6marks

(a)
$$2x - 3 - \frac{6}{x-1} = \frac{(2x-3)(x-1)-6}{x-1} = \frac{2x^2 - 5x - 3}{x-1}$$
 $(x \neq 1)$
(b) Taking $x = \sin(\theta)$ is (a), we get $\frac{2\sin^2(\theta) - 5\sin(\theta) - 3}{\sin(\theta) - 1} = 0$ $\Delta = 25 + 24 = 49$
 $x = \frac{5 \pm 7}{4} = -\frac{1}{2}$ or $3 \Rightarrow \sin(\theta) = -\frac{1}{2} \Rightarrow \boxed{\theta = \frac{\pi}{6}}$ or $\theta = \frac{5\pi}{6}$

Problem 7

/ 7marks

Solve
$$\cos(2x) = 5\cos(x) - 3$$
 for $0 \le x < 2\pi$
 $\Rightarrow \cos^2(x) - \sin^2(x) = 5\cos(x) - 3$
 $\Rightarrow \cos^2(x) - (1 - \cos^2(x)) = 5\cos(x) - 3$
 $\Rightarrow 2\cos^2(x) - 5\cos(x) + 2 = 0$
 $\Rightarrow 2c^2 - 5c + 2 = 0$ with $c = \cos(x)$
 $\Delta = 25 - 16 = 9$ $c = \frac{5 \pm 3}{4} = 2 \text{ or } \frac{1}{2}$ $\Rightarrow \cos(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$ or $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ for $0 \le x \le 2\pi$.