Christmas Examination

Maths AA SL IB$_1$ Part 1

(7 Problems)

Tot: / 42

Tuesday 13 Dec. 2022

Name : _____________

You are not permitted access to any calculator for this paper.

Problem 1 / 5marks

The n$^{\text{th}}$ term of an *arithmetic* sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the n$^{\text{th}}$ term of this sequence is -33 , find the value of n . [2]

(c) Find the common difference, d . [2]

Problem 2 / 5marks

Consider any three consecutive integers, $n-1$, n and $n+1$.

(a) Prove that the *sum* of these three integers is always divisible by 3.

(b) Prove that the *sum* of the <u>squares</u> of these three integers is never divisible by 3.

Problem 3 / 7marks

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + ... + 1$
where $x \neq 0$ and $a, b \in \mathbb{Z}^+$

(a) Show that $b = 21$.

The third term in the expansion is the *mean** of the second term and the fourth term in the expansion. ($\star$: The *mean* of m and n is $\frac{m+n}{2}$)

(b) Find the possible values of x.

Problem 4 / 6marks

(a) Show that the equation $2\cos^2(x) + 5\sin(x) = 4$ may be written in the form $2\sin^2(x) - 5\sin(x) + 2 = 0$.

(b) Hence, solve the equation $2\cos^2(x) + 5\sin(x) = 4$, $0 \leq x \leq 3\pi$.

Problem 5 / 6marks

Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$

Hint : You are supposed to know the exact values of α such that $\cos(\alpha)=\frac{1}{\sqrt{2}}$

Problem 6 / 6marks

(a) Show that $2x-3-\dfrac{6}{x-1}=\dfrac{2x^2-5x-3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta-3-\dfrac{6}{\sin 2\theta-1}=0$ for $0 \leq \theta \leq \pi$, $\theta \neq \dfrac{\pi}{4}$. [5]

Problem 7 / 7marks

Solve $\cos(2x)=5\cos(x)-3$ for $0 \leqslant x < 2\pi$