Maths

IB1 **Examination of June 2019**

Friday 13 June **– Part 1 –** ANSWERS

13 IB's P1 Questions (tot : 80 marks)

Problem 1 [7 marks]

$$\begin{cases} \frac{1}{2}r^2\theta = 12\text{cm}^2 \\ \theta r = 6\text{cm} \end{cases} \Rightarrow \frac{24}{r^2} = \frac{6}{r} \Rightarrow r = 2\text{cm}$$

Problem 2 [8 marks]

(a) $d = u_2 - u_1 = \log_c(pq) - \log_c(p) = \log_c(q)$

(b) if $p = c^2$ and $q = c^3$ then $u_1 = 2\log_c(c)$, $d = 3\log_c(c)$

$$\text{and } \sum_{n=1}^{20} u_n = 20u_1 + \frac{20 \times 19}{2}d = 40\log_c(c) + 190 \times 3\log_c(c) = \boxed{6103\log_c(c)}$$

Problem 3 [6 marks]

$u_1 = -5$ $d = 3$

(a) $u_8 = -5 + 7 \times 3 = \boxed{16}$

(b) $-5 + (n-1)3 = 67 \Rightarrow n = \frac{72}{3} + 1 = \boxed{25}$

Problem 4 [16 marks]

(a) $\log_3(4x+1) + \log_3(x-2) - 2\log_3(3x) = 0 \Rightarrow \frac{(4x+1)(x-2)}{9x^2} = 1$

domain: $x > 2$ $\Rightarrow (4x+1)(x-2) = 9x^2 \Rightarrow 5x^2 + 7x + 2 = 0$

$\boxed{S = \varnothing}$ $\Rightarrow x = -1$ or $x = -\frac{2}{5}$

(b) $\log(x^2 + 2x - 3) - 2\log(x-1) = 2$

domain: $x > 1$ $\Rightarrow \frac{x^2 + 2x - 3}{(x-1)^2} = 100$

$\boxed{S = \{\frac{103}{99}\}}$ $\Rightarrow \frac{(x+3)(x \not- 1)}{(x-1)(x \not- 1)} = 100$ $\Rightarrow x + 3 = 100x - 100$

$\Rightarrow \frac{103}{99}$

(c) $6 + 4^x = 5 \cdot 2^x \Rightarrow y^2 - 5y + 6 = 0$ $y = 2$ or $y = 3$ where $y = 2^x$

$\boxed{S = \{1, \log_2(3)\}}$

Problem 5 [4 marks]

$$\cos(\theta)=\sqrt{1-\frac{1}{9}}=\frac{\sqrt{8}}{3}$$

$$\cos(4\theta)=\cos^2(2\theta)-\sin^2(2\theta)=(\cos^2(\theta)-\sin^2(\theta))^2-(2\sin(\theta)\cos(\theta))^2$$

$$=(\frac{8}{9}-\frac{1}{9})^2-4\frac{1}{9}\times\frac{8}{9}=\frac{49-32}{81}=\boxed{\frac{17}{81}}$$

Problem 6 [4 marks]

Consider the vectors $\boldsymbol{a}=\begin{pmatrix}3\\2p\end{pmatrix}$ and $\boldsymbol{b}=\begin{pmatrix}p+1\\8\end{pmatrix}$.

Find the possible values of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

$$\begin{pmatrix}3\\2p\end{pmatrix}=\lambda\begin{pmatrix}p+1\\8\end{pmatrix}$$

$$2p(p+1)=24\Rightarrow p^2+p-12=0$$

$$\boxed{S=\{-4,3\}}$$

Problem 7 [6 marks]

Find the value of each of the following, giving your answer as an integer

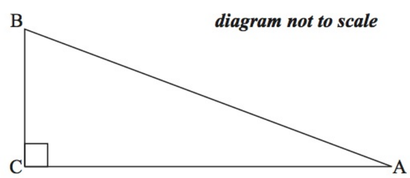
(a) $\log_6(36)=\boxed{2}$

(b) $\log_6(4)+\log_6(9)=\log_6(36)=\boxed{2}$

(c) $\log_6(2)-\log_6(12)=\log_6(\frac{1}{6})=\boxed{-1}$

Problem 8 [5 marks]

The following diagram shows a right-angled triangle, ABC, where $\sin A=\frac{5}{13}$.



(a) Show that $\cos A=\frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

$$\text{Cos}(A)=\sqrt{1-(\frac{5}{13})^2}=\frac{12}{13}\quad\text{and}\cos(2A)=\cos^2(A)-\sin^2(A)=\frac{12^2-5^2}{13^2}=\boxed{\frac{119}{169}}$$

Problem 9 [8 marks]

$u_1 = 1 + k$

$u_2 = s_2 - s_1 = 5 + 3k - (1 + k) = 4 + 2k$

$u_3 = 7 + 4k$

$u_4 = 10 + 8k$ etc....

(b) A general expression for u_n is: $u_n = 1 + (n-1) \times 3 + 2^{(n-1)}k = \boxed{3n - 2 + 2^{n-1}n}$

Problem 10 —–> [3 marks]

(a) (i) correct value 0, or $36 - 12p$

(ii) correct equation which clearly leads to $p = 3$

eg $36 - 12p = 0$, $36 = 12p$

$p = 3$

(b) **METHOD 1**

valid approach

eg $x = -\frac{b}{2a}$

correct working

eg $-\frac{(-6)}{2(3)}$, $x = \frac{6}{6}$

correct answers

eg $x = 1$, $y = 0$; $(1, 0)$

METHOD 2

valid approach

eg $f(x) = 0$, factorisation, completing the square

correct working

eg $x^2 - 2x + 1 = 0$, $(3x - 3)(x - 1)$, $f(x) = 3(x - 1)^2$

correct answers

eg $x = 1$, $y = 0$; $(1, 0)$

METHOD 3

valid approach using derivative

eg $f'(x) = 0$, $6x - 6$

correct equation

eg $6x - 6 = 0$

correct answers

eg $x = 1$, $y = 0$; $(1, 0)$

Problem 11 [6 marks

$\vec{u} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} m \\ 0 \\ n \end{pmatrix}$ $\quad \vec{v} \perp \vec{u} \Rightarrow -3m + n = 0$

$\vec{v}$ unit $\Rightarrow m^2 + n^2 = 1$

$\Rightarrow (3n)^2 + n^2 = 1 \Rightarrow n = \pm\frac{1}{2}$ and $m = \pm\frac{1}{6}$

Problem 12 May17 1.0 Q4 [7 marks]

(a) evidence of choosing the cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS of cosine rule *(A1)*

eg $3^2 + 8^2 - 2\times3\times8\times\cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg $\cos\frac{\pi}{3} = \frac{1}{2}$, $\mathrm{AC}^2 = 9 + 64 - \left(48\times\frac{1}{2}\right)$, $9 + 64 - 24$

correct working clearly leading to answer ***A1***

eg $\mathrm{AC}^2 = 49$, $b = \sqrt{49}$

$\mathrm{AC} = 7$ (cm) ***AG***

Note: Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

(b) correct substitution for semicircle *(A1)*

eg semicircle $= \frac{1}{2}(2\pi\times3.5)$, $\frac{1}{2}\times\pi\times7$, 3.5π

valid approach (seen anywhere) *(M1)*

eg perimeter $= \mathrm{AB} + \mathrm{BC} +$ semicircle, $3 + 8 + \left(\frac{1}{2}\times2\times\pi\times\frac{7}{2}\right)$, $8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi$ $(= 3.5\pi + 11)$ (cm) ***A1***

Maths

IB1 **Examination of June 2019**

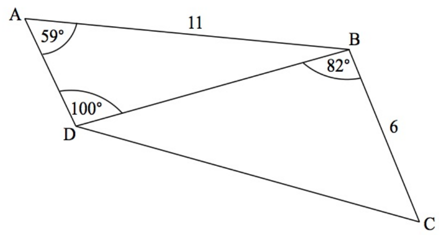
Friday 13 June **– Part 2 –** ANSWERS

6 IB's P2 Questions (tot : 40 marks)

Problem 1 [6 marks]

The following diagram shows quadrilateral ABCD.

diagram not to scale



$AB = 11\,\text{cm}$, $BC = 6\,\text{cm}$, $B\hat{A}D = 59°$, $A\hat{D}B = 100°$, and $C\hat{B}D = 82°$

(a) Find DB. [3]

(b) Find DC. [3]

a) $DB = 11\frac{\sin(59)}{\sin(100)} = \boxed{9.57\text{cm}}$ b) $CD = \sqrt{9.57^2 + 6^2 - 12 \times 9.57\cos(82)} = \boxed{10.56 \text{ cm}}$

Problem 2 [8 marks]

Let us consider the points A(–2; 1), B(2; –2) et C (4; 4).

(a) The lengths in the triangle ABC are AB$=\sqrt{4^2+3^2}=5$, AC$=\sqrt{6^2+3^2}$, BC$=\sqrt{2^2+6^2}$

(b) The angles of triangle ABC are $\hat{A}$ $=\arccos\left(\frac{15}{5\sqrt{45}}\right)=63.43^0$, $\hat{B}$ $\arccos\left(\frac{-6}{\sqrt{25\times 40}}\right)=100.94^0, \ldots$

(c) The area of triangle ABC is $\frac{5\ \sqrt{6^2+3^2}\ \sin(63.43^0)}{2}=15u^2$

Problem 3 [7 marks]

correct substitution into the formula for area of a triangle

eg $15 = \frac{1}{2} \times 8.1 \times 12.3 \times \sin C$

correct working for angle C

eg $\sin C = 0.301114$, $17.5245\ldots$, 0.305860

recognizing that obtuse angle needed

eg 162.475, 2.83573, $\cos C < 0$

evidence of choosing the cosine rule

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$

correct substitution into cosine rule to find c

eg $c^2 = (8.1)^2 + (12.3)^2 - 2(8.1)(12.3)\cos C$

$c = 20.1720$

$8.1 + 12.3 + 20.1720 = 40.5720$

perimeter $= 40.6$

Problem 4 [14 marks]

infinite sum of segments is 2 (seen anywhere)

eg $p + p^2 + p^3 + \ldots = 2$, $\frac{u_1}{1-r} = 2$

recognizing GP

eg ratio is p, $\frac{u_1}{1-r}$, $u_n = u_1 \times r^{n-1}$, $\frac{u_1\left(r^n - 1\right)}{r-1}$

correct substitution into S_∞ formula (may be seen in equation)

eg $\frac{p}{1-p}$

correct equation

eg $\frac{p}{1-p} = 2$, $p = 2 - 2p$

correct working leading to answer

eg $3p = 2$, $2 - 3p = 0$

$p = \frac{2}{3}$ (cm)

(b) recognizing infinite geometric series with squares

eg $k^2 + k^4 + k^6 + \ldots$, $\frac{k^2}{1-k^2}$

correct substitution into $S_\infty = \frac{9}{16}$ (must substitute into formula)

eg $\frac{k^2}{1-k^2} = \frac{9}{16}$

correct working

eg $16k^2 = 9 - 9k^2$, $25k^2 = 9$, $k^2 = \frac{9}{25}$

$k = \frac{3}{5}$ (seen anywhere)

valid approach with segments and CD (may be seen earlier)

eg $r = k$, $S_\infty = b$

correct expression for b in terms of k (may be seen earlier)

eg $b = \frac{k}{1-k}$, $b = \sum_{n=1}^{\infty} k^n$, $b = k + k^2 + k^3 + \ldots$

substituting **their** value of k into **their** formula for b

eg $\frac{\frac{3}{5}}{1-\frac{3}{5}}$, $\frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \frac{3}{2}$

Problem 5 —> [3 marks]

(a) valid approach to find k *(M1)*

eg 8 minutes is half a turn, $k+$ diameter, $k+111=117$

$k=6$ **A1** **N2**

[2 marks]

(b) **METHOD 1**

valid approach *(M1)*

eg $\frac{\max-\min}{2}$, $a=$ radius

$|a|=\frac{117-6}{2}$, 55.5 *(A1)*

$a=-55.5$ **A1** **N2**

METHOD 2

attempt to substitute valid point into equation for f *(M1)*

eg $h(0)=6$, $h(8)=117$

correct equation *(A1)*

eg $6=61.5+a\cos\left(\frac{\pi}{8}\times 0\right)$, $117=61.5+a\cos\left(\frac{\pi}{8}\times 8\right)$, $6=61.5+a$

$a=-55.5$ **A1** **N2**

[3 marks]

(c) valid approach *(M1)*

eg sketch of h and $y=30$, $h=30$, $61.5-55.5\cos\left(\frac{\pi}{8}t\right)=30$, $t=2.46307$, $t=13.5369$

18.4630

$t=18.5$ (minutes) **A2** **N3**

[3 marks]

[Total: 8 marks]

Problem 6 —> [2 marks]

(a) (i) correct value 0, or $36-12p$

(ii) correct equation which clearly leads to $p=3$

eg $36-12p=0$, $36=12p$

$p=3$

(b) **METHOD 1**

valid approach

eg $x=-\frac{b}{2a}$

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correct answers

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eg $x^2-2x+1=0$, $(3x-3)(x-1)$, $f(x)=3(x-1)^2$

correct answers

eg $x=1$, $y=0$; $(1, 0)$

METHOD 3

valid approach using derivative

eg $f'(x)=0$, $6x-6$

correct equation

eg $6x-6=0$

correct answers

eg $x=1$, $y=0$; $(1, 0)$

(c) $x=1$

(d) (i) $a=3$

(ii) $h=1$

(iii) $k=0$

(e) attempt to apply vertical reflection

eg $-f(x)$, $-3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up

eg $-f(x)+6$, vertex $(1, 6)$

transformations performed correctly (in correct order)

eg $-3(x-1)^2+6$, $-3x^2+6x-3+6$

$g(x)=-3x^2+6x+3$