Maths

IB1 **Examination of June 2019**

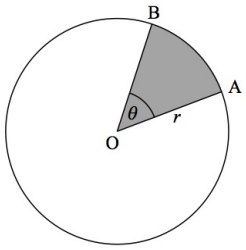
Friday 13 June **– Part 1 –** Name _____

12 IB's P1 Questions (tot : 84 marks)

Problem 1 [7 marks]

The following diagram shows a circle with centre O and radius r cm.

diagram not to scale



The points A and B lie on the circumference of the circle, and $\hat{AOB} = \theta$. The area of the shaded sector AOB is $12\,\text{cm}^2$ and the length of arc AB is $6\,\text{cm}$.

Find the value of r.

Problem 2 [8 marks]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

(a) Show that $d = \log_c(q)$. [2]

(b) Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

Problem 3 [6 marks]

In an arithmetic sequence, $u_1 = -5$ and $d = 3$.

(a) Find u_8. [2]

(b) Find the value of n for which $u_n = 67$. [4]

Problem 4 [16 marks]

Solve the following quations :

(a) $\log_3(4x+1)+\log_3(x-2)-2\log_3(3x)=0$

(b) $\log(x^2+2x-3)-2\log(x-1)=2$

(c) $6+4^x=5\cdot 2^x$

Problem 5 [6 marks]

Assuming $\sin(\theta)=\frac{1}{3}$ et $0<\theta<\frac{\pi}{2}$, find the exact value of $\cos(4\theta)$.

Problem 6 [4 marks]

Consider the vectors $\boldsymbol{a}=\begin{pmatrix}3\\2p\end{pmatrix}$ and $\boldsymbol{b}=\begin{pmatrix}p+1\\8\end{pmatrix}$.

Find the possible values of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

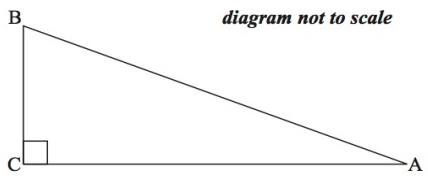
Problem 7 [6 marks]

Find the value of each of the following, giving your answer as an integer

(a) $\log_6(36)$ **(b)** $\log_6(4)+\log_6(9)$ **(c)** $\log_6(2)+\log_6(12)$

Problem 8 [5 marks]

The following diagram shows a right-angled triangle, ABC, where $\sin A=\frac{5}{13}$.



(a) Show that $\cos A=\frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

Problem 9 [8 marks]

The sums of the terms of a sequence follow the pattern

$S_1 = 1+k$, $S_2 = 5+3k$, $S_3 = 12+7k$, $S_4 = 22+15k$, ..., where $k \in \mathbb{Z}$.

(a) Given that $u_1 = 1+k$, find u_2, u_3 and u_4. *[4]*

(b) Find a general expression for u_n. *[4]*

Problem 10 [11 marks]

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

(a) (i) Write down the **value** of the discriminant.

(ii) Hence, show that $p = 3$. *[3]*

The graph of f has its vertex on the x-axis.

(b) Find the coordinates of the vertex of the graph of f. *[4]*

(c) Write down the solution of $f(x) = 0$. *[1]*

(d) The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of

(i) a;

(ii) h;

(iii) k. *[3]*

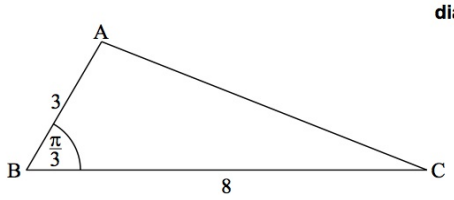
Problem 11 [6 marks]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m, n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n.

Problem 12 May17 1.0 Q4 [7 marks]

The following diagram shows triangle ABC, with $AB = 3\,\text{cm}$, $BC = 8\,\text{cm}$, and $A\hat{B}C = \frac{\pi}{3}$.

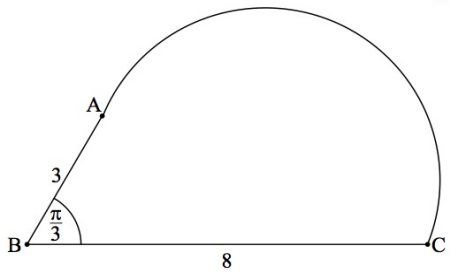
diagram not to scale



(a) Show that $AC = 7\,\text{cm}$. [4]

(b) The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle.

diagram not to scale



Find the exact perimeter of this shape. [3]

Maths

IB1 **Examination of June 2019**

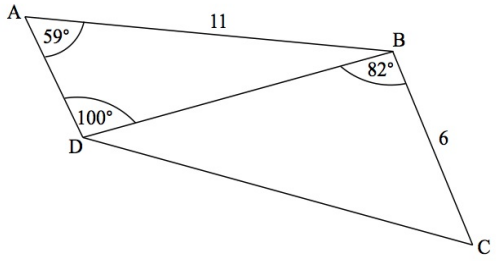
Friday 13 June **– Part 2 –**

6 IB's P2 Questions (tot : 50 marks)

Problem 1 [6 marks]

The following diagram shows quadrilateral ABCD.

diagram not to scale



$AB = 11\,\text{cm}$, $BC = 6\,\text{cm}$, $B\hat{A}D = 59^\circ$, $A\hat{D}B = 100^\circ$, and $C\hat{B}D = 82^\circ$

(a) Find DB. [3]

(b) Find DC. [3]

Problem 2 [8 marks]

Let uc consider the points A(–2; 1), B(2; –2) et C (4; 4).

(a) Find the lengths in the triangle ABC.

(b) Find the valuers of the angles of triangle ABC.

(c) Find the area of triangle ABC

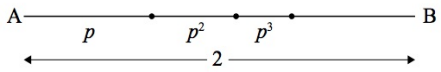
Problem 3 [7 marks]

Triangle ABC has $a = 8.1\,\text{cm}$, $b = 12.3\,\text{cm}$ and area $15\,\text{cm}^2$. Find the largest possible perimeter of triangle ABC.

Problem 4 [14 marks]

(a) The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

diagram not to scale

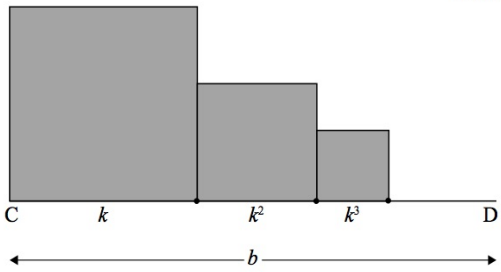


The length of the line segments are p cm, p^2 cm, p^3 cm, …, where $0 < p < 1$.

Show that $p = \frac{2}{3}$. [5]

(b) The following diagram shows [CD], with length b cm, where $b > 1$. Squares with side lengths k cm, k^2 cm, k^3 cm, …, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale

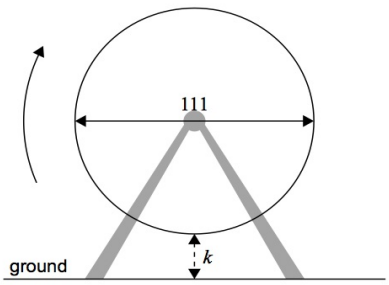


The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b. [9]

Problem 5 [8 marks]

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is k metres above the ground. A seat starts at the bottom of the wheel.

diagram not to scale



The wheel completes one revolution in 16 minutes.

(a) After 8 minutes, the seat is 117 m above the ground. Find k. [2]

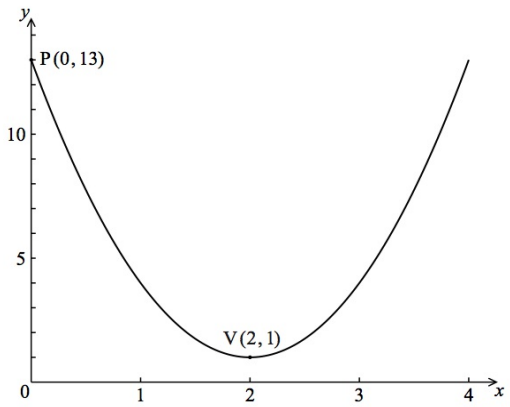
After t minutes, the height of the seat above ground is given by $h(t) = 61.5 + a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

(b) Find the value of a. [3]

(c) Find when the seat is 30 m above the ground for the third time. [3]

Problem 6 [7 marks]

The following diagram shows the graph of a quadratic function f, for $0 \le x \le 4$.



The graph passes through the point $\mathrm{P}(0, 13)$, and its vertex is the point $\mathrm{V}(2, 1)$.

(a) The function can be written in the form $f(x) = a(x-h)^2 + k$.

(i) Write down the value of h and of k.

(ii) Show that $a = 3$. *[4 marks]*

(b) Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. *[3 marks]*