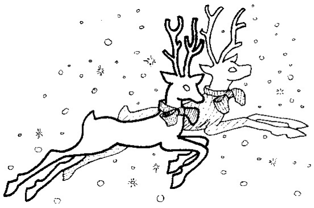

Mathematics HL

IB1 Examination

6 Problems (58 Marks)

Tuesay 10 December 24

(~60min)

Your Name :______________

Problem 1 *[5 marks]*

Solve the equation $3^{(2x \quad 2)} = 3^x \quad 2$

Problem 2 *[9 marks]*

Consider the following sequence of equations:

Eq(1) : $1 \cdot 2 = \frac{1}{3}(1 \cdot 2 \cdot 3)$

Eq(2) : $1 \cdot 2 + 2 \cdot 3 = \frac{1}{3}(2 \cdot 3 \cdot 4)$

Eq(3) : $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 = \frac{1}{3}(3 \cdot 4 \cdot 5)$

1) Verify these equations are correct.

2) Formulate a conjecture (hypothesis) for Eq(4) and verify it.

3) Formulate a general conjecture for Eq(n), for any $n \in \mathbb{N}$

4) Demonstate your conjecture *by induction*.

Problem 3 *[6 marks]*

1) Using Pascal's Triangle (or otherwise), give the expansiont of $(a+b)^5$ and of $(a \quad b)^5$

2) Hence give an expression for $(a+b)^5 + (a \quad b)^5$

3) Show that $(2a)$ is a *factor* of your precedant expression.

Problem 4 *[8 marks]*

Le us consider the sequence $\{u_n\}$ having first terms $2, \quad 1, 3, 13, 35 \ldots.$ (strarting with $n=1$)

1) Is the sequence $\{u_n\}$ *arithmetic*, *geometric* or neither *arithmetic* nor *geometric*?

2) Give the first four terms of the other sequence $\{g_n\}$ with general term is define by: $g_n = \ u_n + a_n$ where $a_n = 3 + 2n$.

3) Show that g_n can be *geometric*.

4) Assuming g_n is *geometric*, find $\sum_{k=5}^{10} g_k$. Help: $\sum_{j=m}^{n} (A_j \pm B_j) = \sum_{j=m}^{n} (A_j \pm B_j) \quad \sum_{j=1}^{m \ 1} (A_j \pm B_j)$

Problem 5 *[13 marks]*

Let us consider the complex numbers $z_1 = 4e^{i\frac{\pi}{6}}$ and $z_2 = \sqrt{2}e^{i\frac{3\pi}{4}}$

1) Write z_1 and z_2 on the form $a + b\,i$?

2) What is the product $z_1^3 \cdot z_2^8$ on the form $a + i\,b$?

3) What is $\left(2\frac{z_2}{z_1}\right)^2$ on the form $a + i\,b$?

4) Show that $\sqrt{i} = \cos\ \frac{\pi}{2}) + i \sin\ \frac{\pi}{2})$

5) Write $\sqrt{z_1^3 \cdot z_2^8}$ on the form $a + bi$

Problem 6 *[17 marks]*

1) Let us consider $z = \frac{i}{2}$.

· What is $|z|$?

· Whas is θ, the *argument* of z ?

. What is z^*, the *conjugate* of z ?

. What are z^2,z^3 and z^4

. Write $\frac{1}{1 \ z}$ on the forme $a + i\,b$

2) Show that, for $|x| < 1$

$$1 + x + x^2 + x^3 + x^4 + \ldots \text{ converges to } \frac{1}{1 \ x}$$

3) We will admit that $1 + z + z^2 + z^3 + z^4 +$ converge the same way for any $z \in \mathbb{C}$, if $|z| < 1$.

. Give an expansion for $\frac{1}{1 \ z}$ with 4 terms.

. Simplify your precedant answer, to give it the form $a + i\,b$

. Consider the limit when the number of terms in the expansion becomes infinite and write the result on the form $a + i\,b$