



## Christmas Examination

### Maths AA SL IB<sub>1</sub> Part 2

( 5 Problems )

Tot: / 54



Tuesday 10 Dec.2024

Name : \_\_\_\_\_

A calculator is permitted for this paper. Any other electronic material allowing access to any AI is strictly prohibited.

#### Question 1

[15 marks]

Let us consider the sequence  $u$ :  $\frac{15}{4}, \frac{3}{2}, \frac{3}{5} \dots$

1) Is it *arithmetic* or *geometric*? [1]

2) What is the first term  $u_n$  smaller than  $\frac{1}{160}$ ? [4]

3) Let us consider the series  $S_n = \sum_{k=1}^n u_k$  [4]

$S_n$  can be written on the form  $\frac{a}{b} \left(1 - \left(\frac{c}{d}\right)^n\right)$ . Find  $a, b, c, d$

4) Give the value of  $\sum_{k=4}^8 u_k$  [4]

5) What gives  $S_n$  if  $n$  is *infinite*? [2]

#### Question 2

[16 marks]

Two friends Amelia and Bill, each set themselves a target of saving \$20 000. They each have \$9000 to invest.

- (a) Amelia invests her \$9000 in an account that offers an interest rate of 7% per annum compounded **annually**.
- (i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.
- (ii) Determine the number of years required for Amelia's investment to reach the target. [5]
- (b) Bill invests his \$9000 in an account that offers an interest rate of  $r\%$  per annum compounded **monthly**, where  $r$  is set to two decimal places.

Find the minimum value of  $r$  needed for Bill to reach the target after 10 years. [3]

- (c) A third friend Chris also wants to reach the \$20 000 target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.
- (i) Show that Chris will never reach the target if his initial deposit is \$9000.
- (ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]

**Question 3**

[9 marks ]

Consider the expansion of  $\left(2x^3 - \frac{k}{x^2}\right)^9$  where  $k > 0$

The coefficient of the term in  $x^{17}$  is 41472

Find  $k$

**Question 4**

[7 marks ]

The expansion of  $(x + h)^8$ , where  $h > 0$ , can be written as  $x^8 + ax^7 + bx^6 + cx^5 + dx^4 + \dots + h^8$ , where  $a, b, c, d, \dots \in \mathbb{R}$ .

(a) Find an expression, in terms of  $h$ , for

(i)  $a$ ;

(ii)  $b$ ;

(iii)  $d$ .

[4]

(b) Given that  $a$ ,  $b$ , and  $d$  are the first three terms of a geometric sequence, find the value of  $h$ .

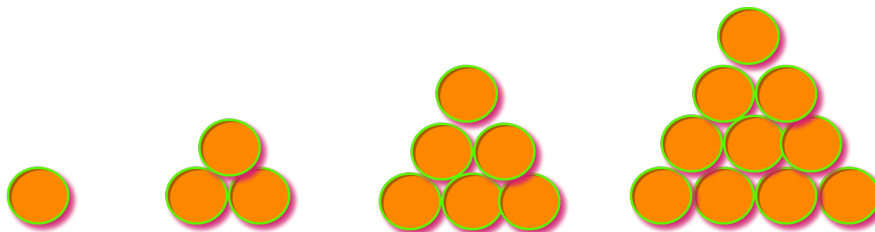
[3]

**Question 5**

[7 marks ]

A child received a box containing 100 small discs for his birthday. He arranges them into groups of increasing size. The figure below shows the first 4 groups.

1) How many complete groups will he be able to form?



2) How many additional boxes (of 100 units) will he need to obtain to double this number of disk groups?