Christmas Examination

Maths AA SL IB_1 Part 1

(7 Problems)

Tot: / 52

Tuesday 10 Dec.2024

ANSWERS

You are not permitted access to any calculator for this paper.

Problem 1 / 7 marks

Solve the following equation:

$$(\log_2(x^2) - \log_2(4)) \cdot \log_2(x) = \log_2(16)$$

$(2\log_2(x) - 2) \cdot \log_2(x) = 4$

$2l^2 - 2l - 4 = 0 \quad \Rightarrow \quad l^2 - l - 2 = 0 \qquad l = 2 \text{ ou } -1 \qquad \boxed{x = 4} \text{ or } \boxed{x = \frac{1}{2}}$

Problem 2 / 6 marks

The nth term of an *arithmetic* sequence is given by $u_n = 15 - 3n$.

(a) The value of the first term is $\boxed{u_1 = 12}$ [1]

(b) $u_n = -33 \Rightarrow 15 - 3n = -33 \Rightarrow$ the value of n is $\boxed{n = 16}$ [2]

(c) The *common difference*, d is $\boxed{-3}$ [2]

Problem 3 (May23 Q4) 7 marks

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

(a) Find the value of p and the value of q. [5]

(b) Find the value of u_5. [2]

(a) $\begin{cases} \text{taking } n = 4 & : 16p - 4q = 40 \\ \text{taking } n = 5 & : 25p - 5q = 65 \end{cases}$ then $4q = 16p - 40$, then $25p - 5(4p - 10) = 65 \quad \Rightarrow 25p - 20p + 50 = 65 \Rightarrow p = \frac{15}{5} = \boxed{3}$

and $q = 12 - 10 = \boxed{2}$

(b) $u_5 = S_5 - S_4 = (3 \times 25 - 2 \times 5) - (3 \times 16 - 2 \times 4) = 65 - 40 = \boxed{25}$

Problem 4 / 8 marks

Let us consider the geometric sequence such that $u_1 = \frac{1}{3}$ and $u_3 = \frac{4}{27}$

(a) $r^2 = \frac{u_3}{u_1} = \frac{4}{27} \times \frac{3}{1} = \frac{4}{9} \Rightarrow r = \boxed{\frac{2}{3}}$

(b) $S_n = u_1 \frac{r^n - 1}{r - 1} = \frac{1}{3}\frac{r^n - 1}{r - 1} = \frac{1}{3}\frac{\left(\frac{2}{3}\right)^n - 1}{1 - \left(\frac{2}{3}\right)} = \boxed{\left(1 - \left(\frac{2}{3}\right)^n\right)}$

(c) $S_\infty = \boxed{1}$

Problem 5 / 8 marks

An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$, where $a > 0$.

$|r| < 1 \Rightarrow \begin{cases} \frac{1}{4}a - 3 < 1 & \text{if } a > 12 \\ \frac{1}{4}a - 3 > -1 & \text{if } a < 12 \end{cases}$

(a) Find the common ratio in terms of a. $\boxed{r = \frac{u_2}{u_1} = \frac{1}{4}a - 3}$ [2]

(b) Find the values of a for which the sum to infinity of the series exists. $\boxed{12 < a < 16}$ or $\boxed{8 < a < 12}$ [3]

(c) Find the value of a when $S_\infty = 76$. $\frac{1}{1 - \left(\frac{1}{4}a - 3\right)} = 76$ $4 - \frac{a}{4} = \frac{1}{76}$ $\boxed{a = 16 - \frac{1}{19} \cong 15.94}$ [3]

Problem 6 (May23 Q6) / 8 marks

The binomial expansion of $(1 + kx)^n$ is given by $1 + 12x + 28k^2x^2 + ... + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

$(1 + kx)^n = 1 + nkx + \frac{n(n-1)}{2!}k^2x^2 + .. + k^nx^n$ then $nk = 12$ and $\frac{n \times (n-1)}{2} = 28$

The second condition implies $n^2 - n - 56 = 0 \Rightarrow n = -7$ or $\boxed{n = 8}$ $(n > 0)$

The first condition implise $8k = 12 \Rightarrow \boxed{k = \frac{3}{2}}$

Problem 7 / 8 marks

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + ... + 1$

where $x \neq 0$ and $a, b \in \mathbb{Z}^+$

(a) $(x+1)^7 = \sum_{k=0}^{7} \binom{7}{k} x^k \cdot 1^{7-k} = \sum_{k=0}^{7} \binom{7}{k} x^k$. We get the value of b taking $k = 5$: $b = \binom{7}{5} = \boxed{21}$

(b) in other term : $\binom{7}{2} x^5 = \frac{\binom{7}{1}x^6 + \binom{7}{3}x^4}{2}$

$\Rightarrow 21x^5 = \frac{7x^6 + 35x^4}{2} \Rightarrow 42x^5 - 7x^6 - 35x^4 = 0 \Rightarrow x^2 - 6x + 5 = 0$

$\Rightarrow (x-5)(x-1) = 0 \Rightarrow \boxed{x \in \{1, 5\}}$