Christmas Examination

Maths AA SL IB_1 Part 1

(7 Problems)

Tot: / 52

Tuesday 10 Dec.2024

Name : _____________

You are not permitted access to any calculator for this paper.

Problem 1 / 7 marks

Solve the following equation:

$$(\log_2(x^2) - \log_2(4)) \cdot \log_2(x) = \log_2(16)$$

Problem 2 / 6 marks

The n^{th} term of an *arithmetic* sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term u_1 [1]

(b) Given that the n^{th} term of this sequence is -33, find the value of n [3]

(c) Find the common difference, d [2]

Problem 3 / 7 marks

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

(a) Find the value of p and the value of q. [5]

(b) Find the value of u_5. [2]

Problem 4 / 8 marks

Let us consider the geometric sequence u such that $u_1 = \frac{1}{3}$ and $u_3 = \frac{4}{27}$

(a) What is the constant ration $r > 0$?

(b) Show that the sum S_7 of the 7 first terms is $3\left(1 - \left(\frac{2}{3}\right)^7\right)$

(c) What is the limit S_∞ of the sum ? (considering $u_1 + u_1 + u_3 + \cdots$ to infinity)

Problem 5 / 8 marks

An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$, where $a > 0$.

(a) Find the common ratio in terms of a. [2]

(b) Find the values of a for which the sum to infinity of the series exists. [3]

(c) Find the value of a when $S_\infty = 76$. [3]

Problem 6 / 8 marks

The binomial expansion of $(1 + kx)^n$ is given by $1 + 12x + 28k^2x^2 + ... + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

Problem 7 / 8 marks

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + ... + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$

(a) Show that b = 21.

(b) The third term in the expansion is the *mean*⋆ of the second term and the fourth term in the expansion, Find the possible values of x.

⋆ *promedio* 平均 : the *mean* of a and b is $\frac{a+b}{2}$