MATHS AA SL

June Exam

PAPER 2

Friday 13 June 2025

Duration : 90 min

7 questions

Total : / 56 marks

Calculator allowed !

ANSWERS

Problem 1

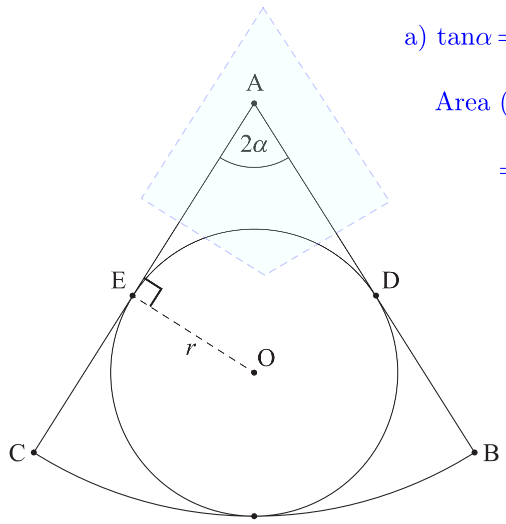
[/10 marks]

The following diagram shows a sector ABC of a circle with centre A. The angle $B\hat{A}C = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $O\hat{E}A = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



a) $\tan\alpha = \frac{r}{AE}$

Area (ADOE) $= 2\frac{AE \times r}{2} = AE \times r$

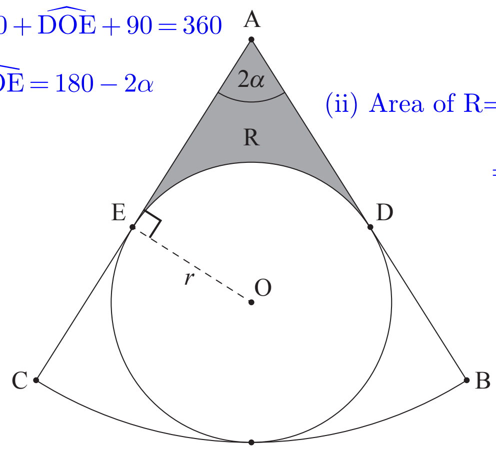
$= \boxed{\frac{r^2}{\tan\alpha}}$

(a) Show that the area of the quadrilateral ADOE is $\dfrac{r^2}{\tan\alpha}$. [4]

(b)

(i) $2\alpha + 90 + \widehat{DOE} + 90 = 360$

$\Rightarrow \widehat{DOE} = 180 - 2\alpha$



(ii) Area of R = Area (ADOE) $- \frac{1}{2}r^2\,\widehat{DOE}$

$= \boxed{\frac{r^2}{\tan\alpha} - \frac{1}{2}r^2(\pi - 2\alpha)}$

(b) (i) Find $D\hat{O}E$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

Problem 2 *[/9 marks]*

Let $f(x) = 2\sin(3x) + 3$ for $x \in \mathbb{R}$

(a) The *average hight* (or $y-$shift) of the curve of equation $y = f(x)$ is $\boxed{3}$ [1]

(b) –The *maximal value* of $f(x)$ is $3+2 = \boxed{5}$ [1]

–The *minimal value* of $f(x)$ is $3-2 = \boxed{1}$ [1]

(c) Its *amplitude* is $\boxed{A=2}$ [1]

(d) Its *periode* is $\boxed{T = \frac{2\pi}{3}}$ [1]

(e) Its *domain* is $\boxed{D_f = \mathbb{R}}$ [1]

(f) The *Range* of $f(x)$ is define as the set of the possible values y such that $y = f(x)$

– A value y that is not in the *range* of $f(x)$ is for example 0, or 7 [1]

– The range of $f(x)$ is $\boxed{R_f = [1,5]}$ [1]

(g) Let $g(x) = 5f(2x) = 5(2\sin(3(2x)) + 3) = 10\sin(6x)) + 15$

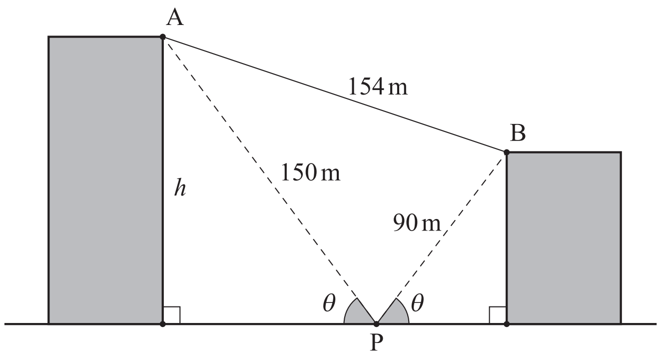
in the form $g(x) = 10\sin(bx) + c$ the values of b and c. are : $\boxed{b = 6 \text{ and } c = 15}$[1]

Problem 3 *[/6 marks]*

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ.

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

(a) Find the measure of $\hat{APB}$. [3]

By cosine rule: $\widehat{APB} = \arccos\left(\frac{154^2 - 150^2 - 90^2}{-2 \times 150 \times 90}\right) = \boxed{72.23^0}$

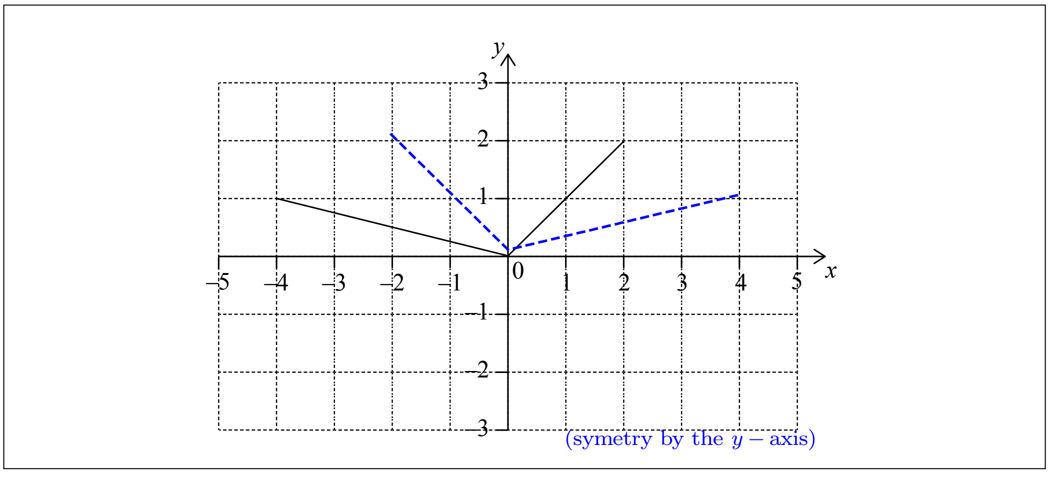
(b) Find the height, h, of the taller building. [3]

Then $\theta = \frac{180 - 72.23}{2} = 52.30^0$

$h = 150\sin(\theta) = \boxed{118.8m}$

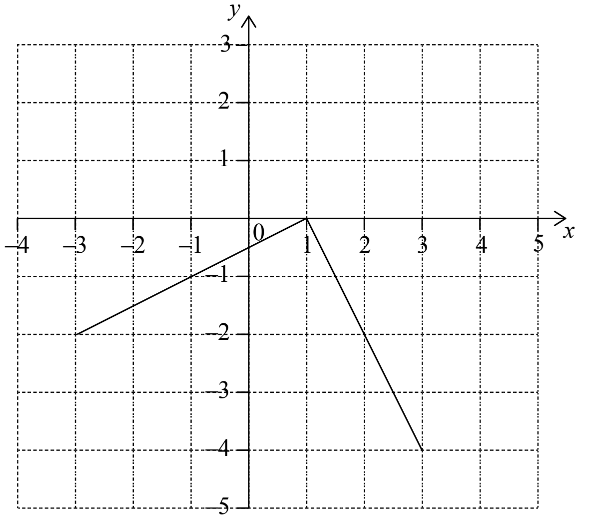
Problem 4 [/6 marks]

The following diagram shows the graph of a function f, for $-4 \leq x \leq 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

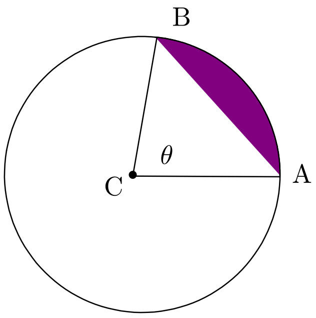
(b) Another function, g, can be written in the form $g(x) = a \times f(x+b)$. The following diagram shows the graph of g.



Write down the value of a and of b. $a = -2$ $b = -1$ [4]

Problem 5 *[/8 marks]*

1) The area of the shaded region is the difference between the area of thetrangle ABC and the area of the circular sector of angle θ.$\mathcal{A} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta)$ $= \frac{1}{2}r^2(\theta - \sin(\theta))$ [4]



where $\theta = \widehat{ACB}$ in rad
C is the center of the circle or r
$r =$ 2cm.

2) : ***solve*** $(\frac{1}{2}2^2(x - \sin(x)) - 3.4, x, 1)$ gives $\underline{2.26717 \text{ rad}}$ $\Rightarrow$ $\boxed{\theta = 129,9^0}$ [4]

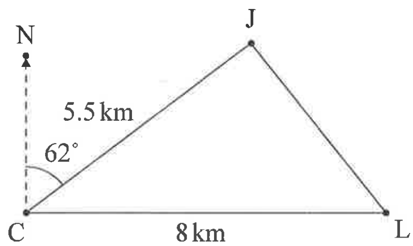
Problem 6 *[/9 marks]*

A lighthouse, L, is located 8 kilometres due East of a coastguard station, C, on a straight stretch of coastline.

The coastguard station sees a Jet Ski, J, on a bearing of 062° and at a distance of 5.5 kilometres. This is shown on the following diagram.

La estación de guardacostas ve una moto acuática J
沿岸警備隊がジェットスキーを発見。
С поста береговой охраны виден гидроцикл (J)

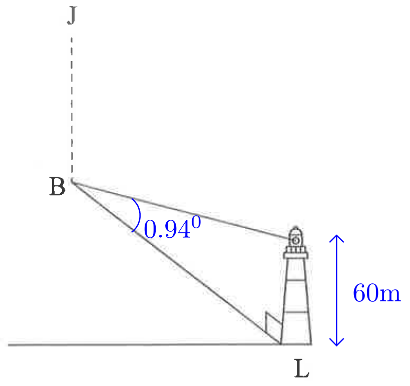
diagram not to scale



(a) Find JL. $JL = \sqrt{5.5^2 + 8^2 - 2 \times 5.5 \times 8 \times \sin(90-62)} = \sqrt{94.25 - 41.32} = 7.28\text{km}$ [4]

While travelling due South, the Jet Ski breaks down at point B, before it reaches the coastline. The position of the Jet Ski at B and the lighthouse are shown in the following diagram.

diagram not to scale



From the top of the 60-metre-tall lighthouse, the angle of depression to the Jet Ski at B, is measured to be 0.94°.

(b) Find BL. $\frac{60}{BL} = \sin(0.94) \quad \Rightarrow BL = 3675m$ [3]

The bearing from the Jet Ski at B to the lighthouse is 121°.

(c) Find the bearing from L to B. $380 - 121 = 750^0$ [2]

Problem 7 *[/8 marks]*

Let $f(x) = \frac{x-2}{2x+1}$ and $g(x) = 1 + \frac{2}{x}$

(a) Find the *domain* of f and the *domain* of g [1]

(b) Give the expression of $(f \circ g)(x)$ [3]

(c) Give the expression of $(g \circ f)(x)$ [2]

(d) Solve $(f \circ g)(x) = (g \circ f)(x)$ [2]

(a) The *domain* of f is: $D_f = \mathbb{R} \setminus \left\{ -\frac{1}{2} \right\}$ and the *domain* of g is: $D_f = \mathbb{R} \setminus \{0\} = \mathbb{R}^*$

(b) $(f \circ g)(x) = \frac{1 + \frac{2}{x} - 2}{2\left(1 + \frac{2}{x}\right) + 1} = \frac{\frac{2}{x} - 1}{\frac{4}{x} + 3} = \frac{2-x}{4+3x}$

(c) $(g \circ f)(x) = 1 + \frac{2}{\frac{x-2}{2x+1}} = 1 + \frac{4x+2}{x-2} = \frac{5x}{x-2}$

(d) $(f \circ g)(x) = (g \circ f)(x) \Leftrightarrow \frac{2-x}{4+3x} = \frac{5x}{x-2} \Leftrightarrow -x^2 + 4x - 4 = 20x + 15x^2$

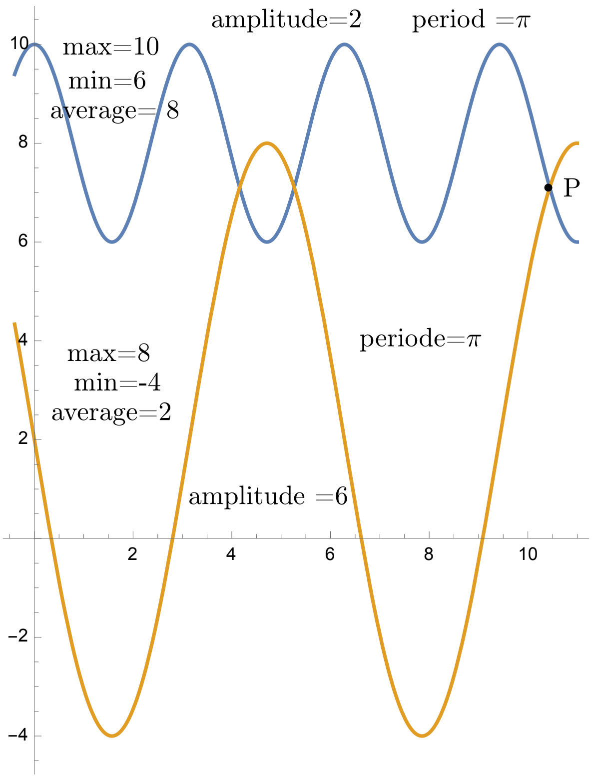
$\Leftrightarrow 16x^2 + 16x + 4 = 0 \quad \Leftrightarrow 4x^2 + 4x + 1 = 0 \quad \Delta = 0 \quad x = -\frac{1}{2}$

Bonus: [max +8]

The picture below show two curves

One has equation $y = \pm \text{A}\cos(kx) + h$ (where A, k, and h are *integers*)

The other one has equation $y = \pm \text{B}\cos(nx) + j$ (where B, n, and j are *integers*)



1) For each curve, find the *minimum*, the *maximum* ,the *range* and the *period*. [+2]

2) Give the values of A, B, k, n, h and j [+3]

3) The *equation* of each curve are : $\boxed{y = 2\cos(2x) + 8 \text{ and } \quad y = -6\sin(x) + 2}$

4) 🖩) Solve$(2\cos(2x) + 6\sin(x) + 6, x, 10)$ $\Rightarrow \boxed{x = 10.4422}$ [+3]