MATHS AA HL

June Exam

PAPER 2

Calculator allowed !

Friday 13 June 2025

Duration : 90 min

7 questions

Total : / 53 marks

ANSWERS

Problem 1 *(HL nov 2023)* [/9 marks]

Three points are given by $\mathrm{A}(0, p, 2)$, $\mathrm{B}(1, 1, 1)$ and $\mathrm{C}(p, 0, 4)$, where p is a positive constant.

(a) Show that $\vec{\mathrm{AB}} \times \vec{\mathrm{AC}} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1-p & -1 \\ p & -p & 2 \end{vmatrix} = (2-3p)\vec{i} + \cdots$ [4]

(b) Hence, find the smallest possible value of $\left|\vec{\mathrm{AB}} \times \vec{\mathrm{AC}}\right|^2$ $=(2-3p)^2+(-2-p)^2+(p^2-2p)^2$ $=p^4-4p^3+14p^2-8p+8$ [3]

(c) Hence, find the smallest possible area of triangle ABC. $\boxed{\frac{\sqrt{6.75}}{2} \simeq 1.3}$ min for $p = 0.3264\ldots$ [2]

Problem 2 *(HL may 2025 !)* [/7 marks]

Consider the planes Π_1, Π_2 and Π_3 with the following equations.

$$\begin{aligned} \Pi_1 : &\quad x - y + z = -4 \\ \Pi_2 : &\quad 2x + y - z = -1 \\ \Pi_3 : &\quad -x + y + kz = -3 \end{aligned}$$

$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & 1 & k \end{vmatrix} = 3k+3$

Where $k \in \mathbb{R}$.

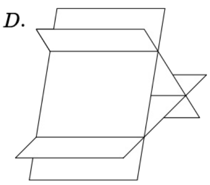
The system of equations that represents the three planes is inconsistent.

Inconsistant means *singular* (det=0)

(a) (i) Find k. $\boxed{k=-1}$

(ii) Describe the geometrical relationship of the three planes. The 3 planes have no commun intersection [3]

L is the line of intersection between Π_1 and Π_2 and it crosses the $xy-$plane at point D.



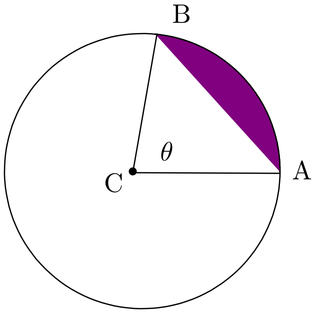
(b) (i) Verify that the vector equation of L can be written as

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 3\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ pass by $\left(-\frac{5}{3}, 0, \frac{-7}{3}\right)$ $\quad \boldsymbol{r} = \begin{pmatrix} -\frac{5}{3} \\ 0 \\ -\frac{7}{3} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(ii) Hence find the coordinates of point D. $\frac{-7}{3} + \lambda = 0 \Rightarrow \lambda = \frac{7}{3}$ $\Rightarrow \boxed{D\colon \left(-\frac{5}{3}, \frac{7}{3}, 0\right)}$ [4]

Problem 3 [/8 marks]

1) The area of the shaded region is the difference between the area of thetrangle ABC and the area of the circular sector of angle θ.$\mathcal{A} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta) = \frac{1}{2}r^2(\theta - \sin(\theta))$ [4]



where $\theta = \widehat{\mathrm{ACB}}$ in rad
C is the center of the circle or r
$r =$ 2cm.

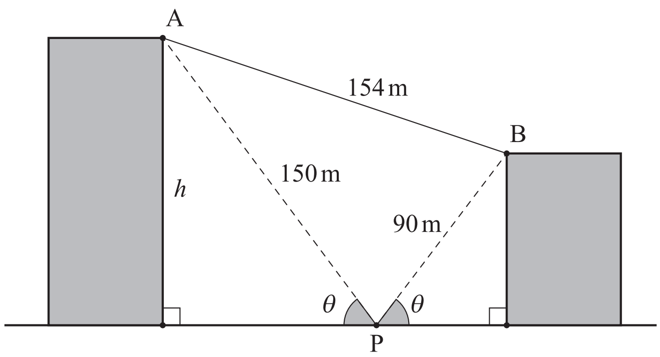
2) : ***solve*** $(\frac{1}{2}2^2(x - \sin(x)) - 3.4, x, 1)$ gives 2.26717 rad $\Rightarrow$ $\boxed{\theta = 129, 9^0}$ [4]

Problem 4 (SL) [/6 marks]

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ.

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

By cosine rule: $\widehat{\mathrm{APB}} = \arccos\left(\frac{154^2 - 150^2 - 90^2}{-2 \times 150 \times 90}\right) = \boxed{72.23^0}$

(a) Find the measure of $\mathrm{A\hat{P}B}$. [3]

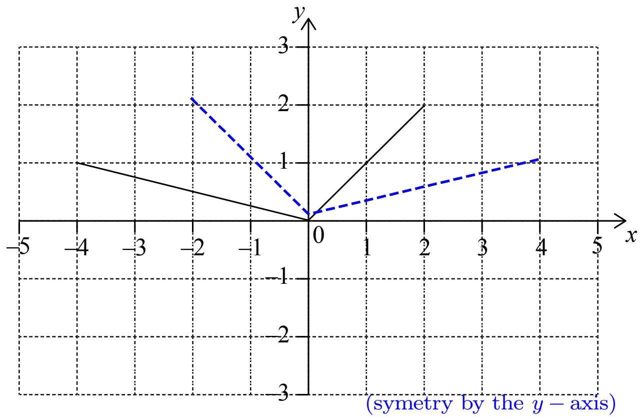
Then $\theta = \frac{180 - 72.23}{2} = 52.30^0$

(b) Find the height, h, of the taller building. [3]

$h = 150\sin(\theta) = \boxed{118.8m}$

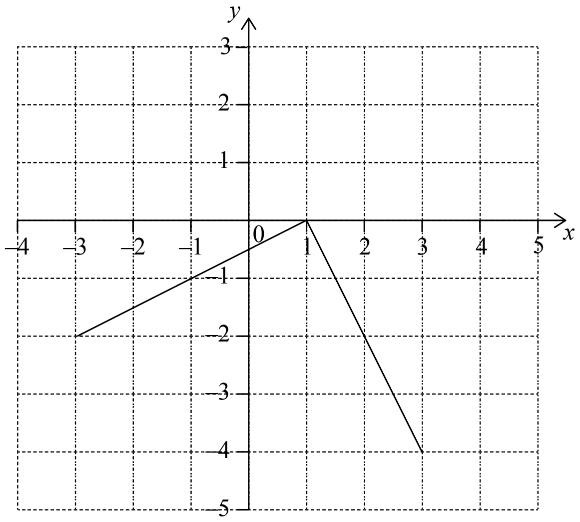
Problem 5 (SL) [/6 marks]

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, g, can be written in the form $g(x) = a \times f(x+b)$. The following diagram shows the graph of g.



Write down the value of a and of b. $a = -2$ $b = -1$ [4]

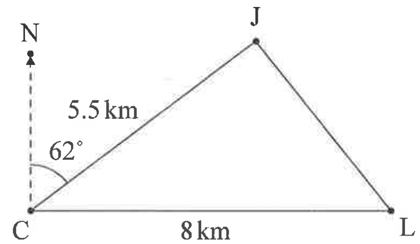
Problem 6 (SL) [/9 *marks*]

A lighthouse, L, is located 8 kilometres due East of a coastguard station, C, on a straight stretch of coastline.

The coastguard station sees a Jet Ski, J, on a bearing of $062°$ and at a distance of 5.5 kilometres. This is shown on the following diagram.

La estación de guardacostas ve una moto acuática J
沿岸警備隊がジェットスキーを発見。
С поста береговой охраны виден гидроцикл (J)

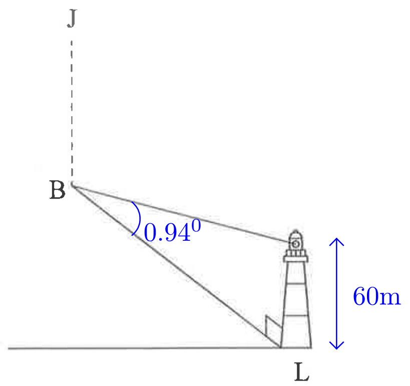
diagram not to scale



(a) Find JL. $JL = \sqrt{5.5^2 + 8^2 - 2 \times 5.5 \times 8 \times \sin(90-62)} = \sqrt{94.25 - 41.32} = 7.28\text{km}$ [4]

While travelling due South, the Jet Ski breaks down at point B, before it reaches the coastline. The position of the Jet Ski at B and the lighthouse are shown in the following diagram.

diagram not to scale



From the top of the 60-metre-tall lighthouse, the angle of depression to the Jet Ski at B, is measured to be $0.94°$.

(b) Find BL. $\frac{60}{BL} = \sin(0.94) \Rightarrow \boxed{BL = 3675m}$ [3]

The bearing from the Jet Ski at B to the lighthouse is $121°$.

(c) Find the bearing from L to B. $380 - 121 = 750^0$ [2]

Problem 7 *(SL)* **[/8 marks]**

Let $f(x) = \frac{x-2}{2x+1}$ and $g(x) = 1 + \frac{2}{x}$

(a) Find the *domain* of f and the *domain* of g [1]

(b) Give the expression of $(f \circ g)(x)$ [3]

(c) Give the expression of $(g \circ f)(x)$ [2]

(d) Solve $(f \circ g)(x) = (g \circ f)(x)$ [2]

(a) The *domain* of f is: $\boxed{D_f = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}}$ and the *domain* of g is: $\boxed{D_f = \mathbb{R} \setminus \{0\} = \mathbb{R}^*}$

(b) $\boxed{(f \circ g)(x) = \frac{1 + \frac{2}{x} - 2}{2\left(1 + \frac{2}{x}\right) + 1} = \frac{\frac{2}{x} - 1}{\frac{4}{x} + 3} = \frac{2 - x}{4 + 3x}}$

(c) $\boxed{(g \circ f)(x) = 1 + \frac{2}{\frac{x-2}{2x+1}} = 1 + \frac{4x+2}{x-2} = \frac{5x}{x-2}}$

(d) $(f \circ g)(x) = (g \circ f)(x) \Leftrightarrow \frac{2-x}{4+3x} = \frac{5x}{x-2} \Leftrightarrow -x^2 + 4x - 4 = 20x + 15x^2$

$\Leftrightarrow 16x^2 + 16x + 4 = 0 \quad \Leftrightarrow 4x^2 + 4x + 1 = 0 \quad \Delta = 0 \quad \boxed{x = -\frac{1}{2}}$

Bonus: [max +7]

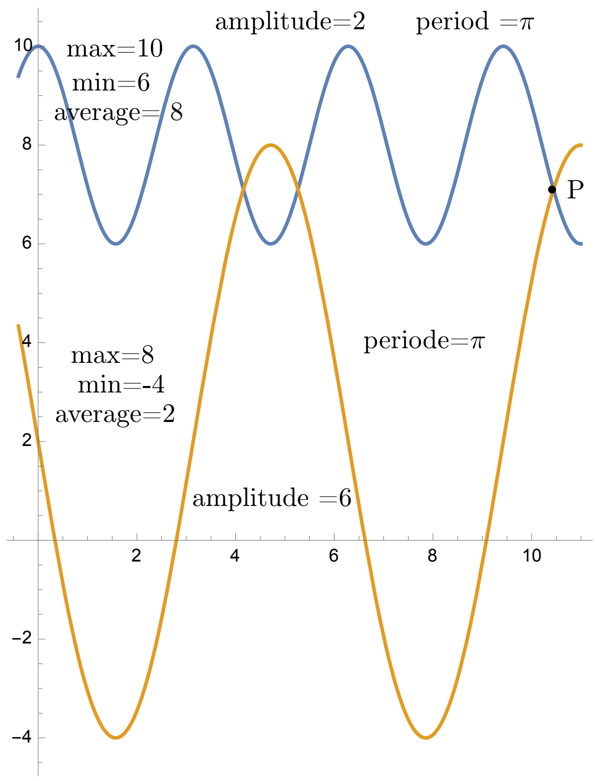
The picture below show two curves

One has equation $y = \pm A\cos(kx) + h$ (where $A, k,$ and h are *integers*)

The other one has equation $y = \pm B\cos(nx) + j$ (where $B, n,$ and j are *integers*)

1) Find the equation for each of the two curve [+4]

2) Using *solve* (🖩), find the x-coordinate of P the point of intersection shown on the picture. (you will have chose a smart *guess value*) [+3]



1) For each curve, find the *minimum*, the *maximum* ,the *range* and the *period*. [+2]

2) Give the values of A, B, k, n, h and j [+3]

3) The *equation* of each curve are : $\boxed{y = 2\cos(2x) + 8 \text{ and } \quad y = -6\sin(x) + 2}$

4) 🖩) Solve$(2\cos(2x) + 6\sin(x) + 6, x, 10)$ $\Rightarrow \boxed{x = 10.4422}$ [+3]