
# MATHS AA HL

**June Exam**

PAPER 2

Friday 13 June 2025

Duration : 90 min

7 questions

Total : / 53 marks

Calculator allowed !

Nom/Name _____

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## Problem 1 *(HL nov 2023)* [ /9 marks ]

Three points are given by $A(0, p, 2)$, $B(1, 1, 1)$ and $C(p, 0, 4)$, where $p$ is a positive constant.

(a) Show that $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$. [4]

(b) Hence, find the smallest possible value of $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|^2$. [3]

(c) Hence, find the smallest possible area of triangle $ABC$. [2]

## Problem 2 *(HL may 2025 ! )* [ /7 marks ]

Consider the planes $\Pi_1$, $\Pi_2$ and $\Pi_3$ with the following equations.

$$\Pi_1 : \quad x - y + z = -4$$
$$\Pi_2 : \quad 2x + y - z = -1$$
$$\Pi_3 : \quad -x + y + kz = -3$$

Where $k \in \mathbb{R}$.

The system of equations that represents the three planes is inconsistent.

*Inconsistant* means *singular* (det=0)

(a) (i) Find $k$.

(ii) Describe the geometrical relationship of the three planes. [3]

$L$ is the line of intersection between $\Pi_1$ and $\Pi_2$ and it crosses the $xy-$plane at point $D$.

(b) (i) Verify that the vector equation of $L$ can be written as

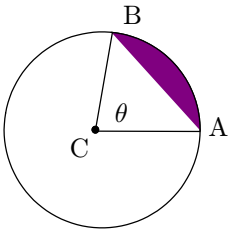
$$\boldsymbol{r} = \begin{pmatrix} -\frac{5}{3} \\ 0 \\ -\frac{7}{3} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

(ii) Hence find the coordinates of point $D$. [4]

**Problem 3** (SL) *[ /8 marks ]*

**1)** Show that the area of the shaded region is given by the formula

$\mathcal{A} = \frac{1}{2}r^2(\theta - \sin(\theta))$ [4]




where $\theta = \widehat{\text{ACB}}$ in rad
C is the center of the circle or $r$
$r =$ 2cm.

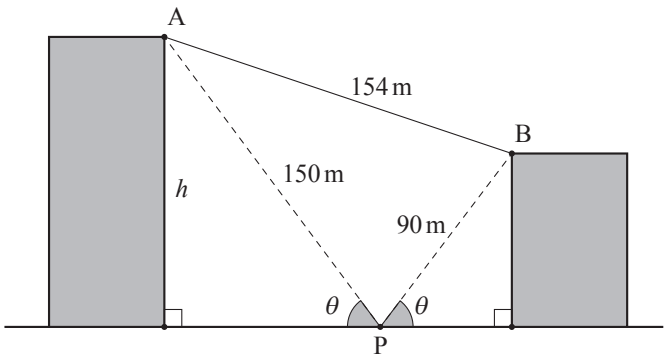
**2)** Using *solve* (), find the value of $\theta$ (in radian and in degree) for having $\mathcal{A} = 3.4\text{cm}^2$ [4]

**Problem 4** (SL) *[ /6 marks ]*

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is $\theta$.

**diagram not to scale**




The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

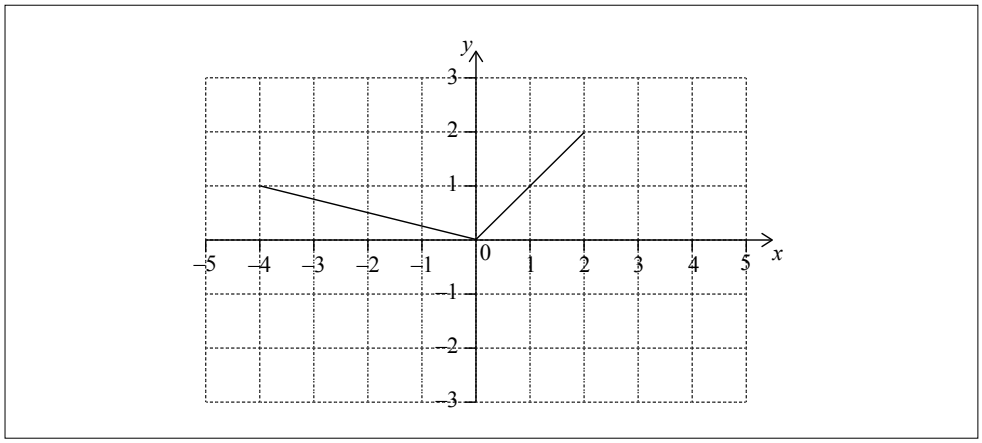
The distance between A and B is 154 metres.

(a) Find the measure of $\text{A}\hat{\text{P}}\text{B}$. [3]

(b) Find the height, $h$, of the taller building. [3]

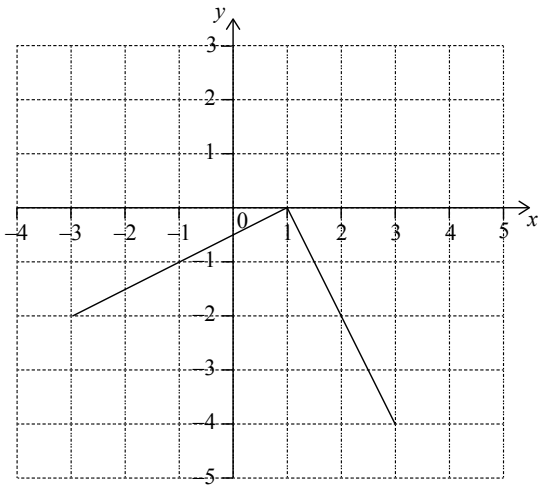
**Problem 5** (SL) *[ /6 marks ]*

The following diagram shows the graph of a function $f$, for $-4 \le x \le 2$.




(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, $g$, can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of $g$.




Write down the value of $a$ and of $b$. [4]

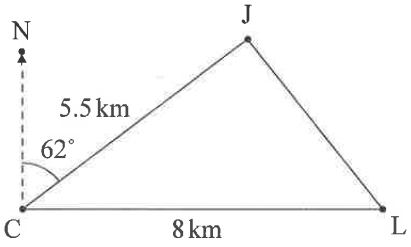
**Problem 6** (SL) / /9 marks /

A lighthouse, L, is located 8 kilometres due East of a coastguard station, C, on a straight stretch of coastline.

The coastguard station sees a Jet Ski, J, on a bearing of 062° and at a distance of 5.5 kilometres. This is shown on the following diagram.

| La estación de guardacostas ve una moto acuática J |
|---|
| 沿岸警備隊がジェットスキーを発見。 |
| С поста береговой охраны виден гидроцикл (J) |

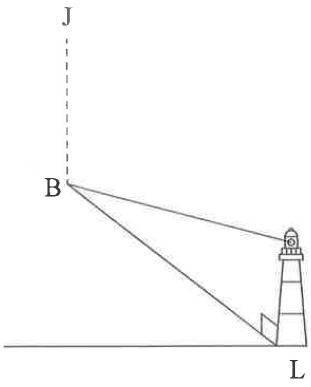
**diagram not to scale**




(a) Find JL. [4]

While travelling due South, the Jet Ski breaks down at point B, before it reaches the coastline. The position of the Jet Ski at B and the lighthouse are shown in the following diagram.

**diagram not to scale**




From the top of the 60-metre-tall lighthouse, the angle of depression to the Jet Ski at B, is measured to be 0.94°.

(b) Find BL. [3]

The bearing from the Jet Ski at B to the lighthouse is 121°.

(c) Find the bearing from L to B. [2]

**Problem 7** *(SL)* **[ /8 marks ]**

Let $f(x)=\frac{x-2}{2x+1}$ and $g(x)=1+\frac{2}{x}$

**(a)** Find the *domain* of $f$ and the *domain* of $g$ [1]

**(b)** Give the expression of $(f\circ g)(x)$ [3]

**(c)** Give the expression of $(g\circ f)(x)$ [2]

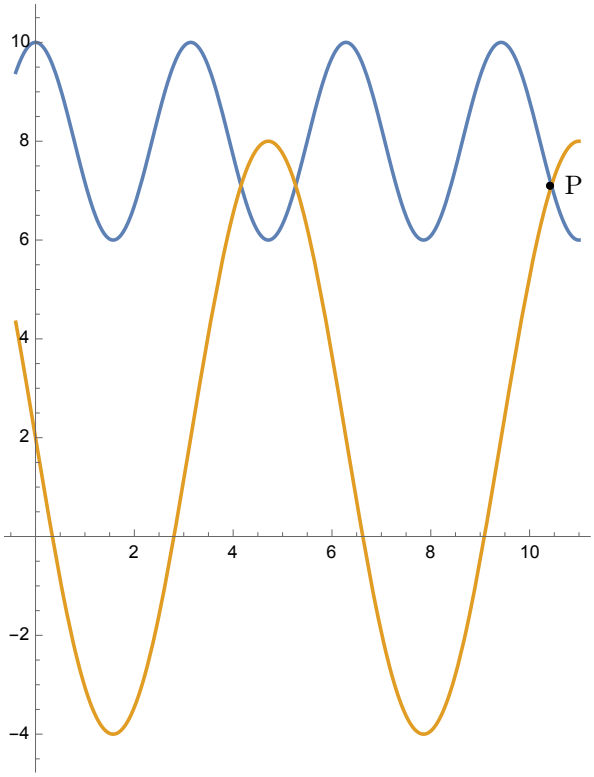
**(b)** Solve $(f\circ g)(x)=(g\circ f)(x)$ [2]

**Bonus:** [ max +7 ]

The picture below show two curves

One has equation $y=\pm A\cos(kx)+h$ (where $A, k,$ and $h$ are *integers*)

The other one has equation $y=\pm B\cos(nx)+j$ (where $B, n,$ and $j$ are *integers*)




**1)** Find the equation for each of the two curve [+4]

**2)** Using *solve* (), find the $x$-coordinate of P the point of intersection shown on the picture. (you will have chose a smart *guess value*) [+3]