MATHS AA HL

June Exam

PAPER I

Friday 13 June 2025

Duration : 90 min

7 questions

Total : / 50 marks

Calculator not allowed !

ANSWERS

Problem 1 *(HL may 2025 !)* *[/5 marks]*

$12\ \log_x(2) = 12\,\frac{\log_2(2)}{\log_2(x)} = \frac{12}{\log_2(x)}$ [1]

Hence solve $\log_2(x) = 8 - 12\ \log_x(2)$ $\Leftrightarrow \log_2(x) = 8 - \frac{12}{\log_2(x)}$

$\Leftrightarrow \log_2^2(x) - 8\log_2(x) + 12 = 0$

$\Delta = 64 - 4\times 12 = 16$ $\Rightarrow \log_2(x) = \frac{8\pm 4}{2} = 2 \text{ or } 6$ $\Rightarrow x = 4 \text{ or } x = 64$

Problem 2 *(HL may 2025 !)* *[/7 marks]*

(a) $4 - 3\cos(2x) = 4 - 3(1 - 2\sin^{2(x)}) = 6\sin^2(x) + 1$

(b) Hence $4 - 3\cos(4\theta + \frac{2\pi}{3}) - 9\sin(2\theta + \frac{\pi}{3}) = -2$, for $0 \le \theta \le \pi$

$\Leftrightarrow 6\sin^2(4\theta + \frac{2\pi}{3}) + 1 - 9\sin(2\theta + \frac{\pi}{3}) = -2$, for $0 \le \theta \le \pi$

$\Leftrightarrow 6\sin^2(4\theta + \frac{2\pi}{3}) - 9\sin(2\theta + \frac{\pi}{3}) + 3 = 0$, for $0 \le \theta \le \pi$

$\Delta = (-9)^2 - 4\times 6\times 3 = 9$ $\Rightarrow \sin(2\theta + \frac{\pi}{3}) = \frac{9\pm 3}{12} = \frac{1}{2} \text{ or } 1$, for $0 \le \theta \le \pi$

$\Rightarrow 2\theta + \frac{\pi}{3} = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}, \text{ or } \frac{\pi}{2}$

$\Rightarrow 2\theta = 0 \text{ or } \frac{\pi}{3}, \text{ or } \frac{\pi}{6}$

$\Rightarrow \boxed{\theta \in \{0, \frac{\pi}{6}, \frac{\pi}{4}\}}$

Problem 3 *(HL may 2025 !)b* *[/5 marks]*

(a) (i) $2\binom{n}{r} = \binom{n}{r+1} \Leftrightarrow \frac{2n!}{(n-r)!r!} = \frac{n!}{(n-r-1)!(r+1)!}$

$\Leftrightarrow \frac{2\,(n-r-1)!}{(n-r)!} = \frac{r!}{(r+1)!} \Leftrightarrow \frac{2}{(n-r)} = \frac{1}{(r+1)} \Leftrightarrow 2\,(r+1) = n - r$

$\Leftrightarrow n = 3r + 2$

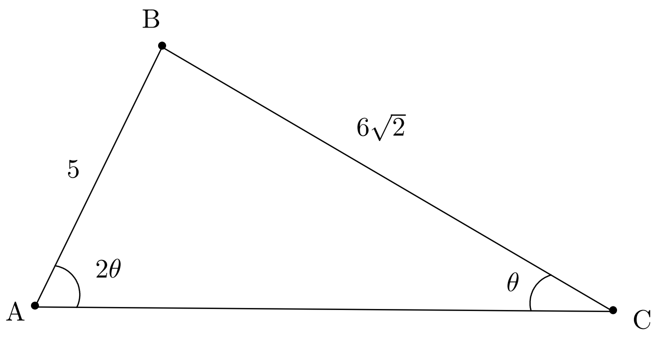
(ii) $7\binom{n}{r-1} = 2\binom{n}{r} \Leftrightarrow \frac{7n!}{(n-r+1)!(r-1)!} = \frac{2n!}{(n-r)!r!}$

$\Leftrightarrow \frac{7}{2} = \frac{(n-r+1)!(r-1)!}{(n-r)!r!} = \frac{n-r+1}{r} \Leftrightarrow 7r = 2n - 2r + 2 \Leftrightarrow 9r - 2 = 2n$

(b) The question was missing !

Problem 4 *(SL may 2025 !)* [/7 marks]

The following diagram shows a non-right angled triangle ABC



$$AB = 5, \quad BC = 6\sqrt{2} \quad \widehat{ACB} = \theta \quad \text{and} \quad \widehat{BAB} = 2\theta \qquad \text{where } 0 < \theta < \tfrac{\pi}{2}.$$

(a) Using the sine rule: $\frac{\sin(\theta)}{5} = \frac{\sin(2\theta)}{6\sqrt{2}} \Rightarrow 6\sqrt{2}\sin(\theta) = 10\cos(\theta)\sin(\theta) \Rightarrow \boxed{\cos\,\theta = \frac{3\sqrt{2}}{5}}$

(b) Hence $\sin(\theta) = \sqrt{1 - \left(\frac{3\sqrt{2}}{5}\right)^2} = \sqrt{1 - \frac{18}{25}} = \boxed{\frac{\sqrt{7}}{5}}$

Point D is located on [AC] such that the area of triangle BCD is $6\sqrt{14}$.

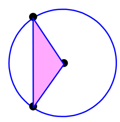
(c) Area of triangle BCD$=\frac{1}{2}\text{DC} \times \text{BC}\,\sin(\theta) \Rightarrow 6\sqrt{14} = \frac{1}{2}\text{DC} \times 6\sqrt{2} \Rightarrow \boxed{\text{DC} = 2\sqrt{7}}$

Problem 5 *(HL Nov.2020)* [/5 marks]

Consider the complex numbers $z_1 = \cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}$ and $z_2 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$.

(a) (i) Find $\frac{z_1}{z_2} = \frac{\text{cis}\left(\frac{11}{12}\pi\right)}{\text{cis}\left(\frac{1}{6}\pi\right)} = \text{cis}\left(\frac{11}{12}\pi - \frac{1}{6}\pi\right) = \text{cis}\left(\frac{9}{12}\pi\right) = \boxed{\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}(-1+i)}$

(ii) Find $\frac{z_2}{z_1} = \left(\frac{z_1}{z_2}\right)^{-1} = \left(\text{cis}\left(\frac{9}{12}\pi\right)\right)^{-1} = \text{cis}\left(-\frac{3}{4}\pi\right) = \boxed{\frac{\sqrt{2}}{2}(-1-i)}$ [3]



(b) 0, $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$ are represented by three points O, A and B respectively on an Argand diagram. Determine the area of the triangle OAB. $\boxed{A = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\sqrt{2} = \frac{1}{2}}$ [2]

Problem 6 *(SL may 2025 !)* *[/13 marks]*

(a) $b = \frac{2\pi}{T} = \boxed{\frac{\pi}{6}}$

(b) $a = \boxed{2.3m}$

(c) $d = \frac{2.2+6.8}{2} = \boxed{4.5m}$

(d) $H(4{:}30) = 6.8 \Rightarrow 2.3\sin\left(\frac{\pi}{6}(4.5-c)\right) + 4.5 = 6.8 \Rightarrow \sin\left(\frac{\pi}{6}(4.5-c)\right) = 1$

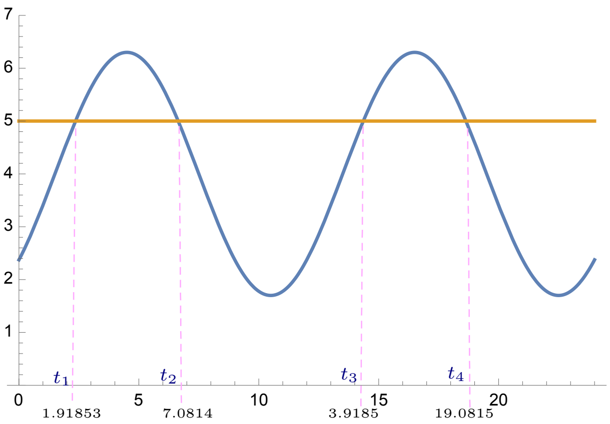
$\Rightarrow \frac{\pi}{6}(4.5-c) = \frac{\pi}{2} \Rightarrow (4.5-c) = 3$

$\Rightarrow \boxed{c = \frac{3}{2}}$

(e) $H(12) = 2.3\sin\left(\frac{\pi}{6}\left(12-\frac{3}{2}\right)\right) + 4.5 = 2.3\sin\left(\frac{\pi}{6}\left(\frac{21}{2}\right)\right) + 4.5 = 2.3\sin\left(\frac{7\pi}{4}\right) + 4.5 = \boxed{2.874m}$

(f) As the period is 12hours, we first find the solutions of $H(t) = h$

with $H(t) = 2.3\sin\left(\frac{\pi}{6}\left(t-\frac{3}{2}\right)\right) + 4.5$ and $h = 5$

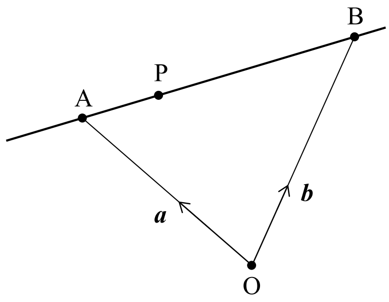


We find $t_1 = 2.35905h$, $t_2 = 6.64095h$, $t3 = 14.359h$, $t_4 = 18.641h$

Then $H(t) > h$ for $t \in]t_1, t_2[\cup]t_3, t_4[$ that is during $t_2 - t_1 + t_4 - t_3 = 2 \times 5.16294 = \boxed{10.326h}$

Problem 7 *(HL Nov 2024)* [/8 marks]

The following diagram shows two points A and B such that $\vec{OA} = \boldsymbol{a}$, $\vec{OB} = \boldsymbol{b}$.



The point P lies on (AB) so that $\vec{AP} = \lambda \vec{AB}$ where $0 < \lambda < 1$.

(a) Show that $\vec{OP} = (1-\lambda)\boldsymbol{a} + \lambda\boldsymbol{b}$. [1]

It is given that $|\boldsymbol{a}| = 1$, $|\boldsymbol{b}| = 2$ and $\boldsymbol{a}\cdot\boldsymbol{b} = \frac{1}{4}$.

(b) In the case that $\vec{OP}$ is perpendicular to $\vec{AB}$, find the value of λ. [7]

(a) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \vec{a} + \lambda\overrightarrow{AB} = \vec{a} + \lambda(\overrightarrow{OB} - \overrightarrow{OA}) = \vec{a} + \lambda(\vec{b} - \vec{a}) = (1-\lambda)\vec{a} + \lambda\vec{b}$

(b) $\overrightarrow{OP} \perp \overrightarrow{AB} \Leftrightarrow \overrightarrow{OP}\cdot\overrightarrow{AB} = 0 \Leftrightarrow ((1-\lambda)\vec{a} + \lambda\vec{b})\cdot(\vec{b} - \vec{a}) = 0$

$\Leftrightarrow (1-\lambda)\vec{a}\cdot\vec{b} + \lambda\vec{b}\cdot\vec{b} - (1-\lambda)\vec{a}\cdot\vec{a} - \lambda\vec{a}\cdot\vec{b} = 0$

$\Leftrightarrow (1-2\lambda)\vec{a}\cdot\vec{b} + \lambda|\vec{b}|^2 - (1-\lambda)|\vec{a}|^2 = 0$

$\Leftrightarrow (1-2\lambda)\frac{1}{4} + \lambda 2^2 - (1-\lambda) = 0$

$\Leftrightarrow \frac{1}{4} - \frac{1}{2}\lambda + 4\lambda - 1 + \lambda = 0 \quad \Leftrightarrow \frac{-3}{4} + \frac{9}{2}\lambda = 0 \Rightarrow \boxed{\lambda = \frac{1}{6}}$