MATHS AA HL

June Exam

PAPER I

Friday 13 June 2025

Duration : 90 min

7 questions

Total : / 50 marks

Calculator not allowed !

Nom/Name ______________

Problem 1 *(HL may 2025 !)* *[/5 marks]*

Show that $12\ \log_x(2) = \frac{12}{\log_2(x)}$ [1]

Hence solve $\log_2(x) = 8 - 12\ \log_x(2)$ [4]

Problem 2 *(HL may 2025 !)* *[/7 marks]*

(a) Show that $4 - 3\cos(2x) = 6\sin^2(x) + 1$ [1]

(b) Hence or otherwise solve $4 - 3\cos(4\theta + \frac{2\pi}{3}) - 9\sin(2\theta + \frac{\pi}{3}) = -2$, for $0 \leq \theta \leq \pi$ [6]

Problem 3 *(HL may 2025 !)b* *[/5 marks]*

(a) (i) Consider the following equation $2\binom{n}{r} = \binom{n}{r+1}$.

Show that it can be written as $3r + 2 = n$.

(ii) Now consider the following equation $7\binom{n}{r-1} = 2\binom{n}{r}$.

Show that it can be written as $9r - 2 = 2n$.

~~Consider the expansion~~

This part of question was missing; It will be discussed in lass

$$(1+x)^n = 1 + a_1 x + \ldots + a_{k-1}x^{k-1} + a_k x^k + a_{k+1}x^{k+1} + \ldots + x^n$$

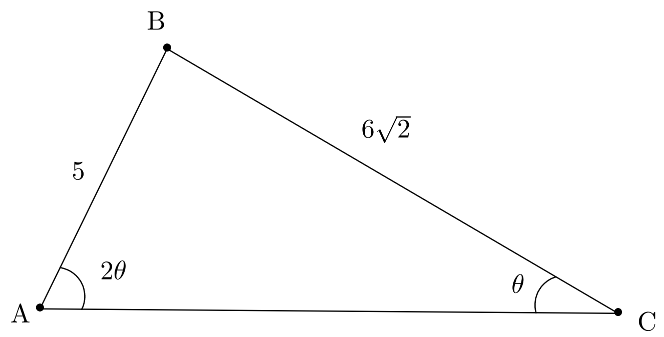
Where $a_i \in \mathbb{Q}$ and $k \in \mathbb{Z}$.

The coefficients of three consecutive terms of the expansion are such that

$$7 \times a_{k-1} = 2 \times a_k \quad \text{and} \quad 14 \times a_k = 7 \times a_{k+1}$$

Problem 4 *(SL may 2025 !)* [/7 marks]

The following diagram shows a non-right angled triangle ABC



$\mathrm{AB}=5$, $\mathrm{BC}=6\sqrt{2}$ $\widehat{\mathrm{ACB}}=\theta$ and $\widehat{\mathrm{BAB}}=2\theta$ where $0<\theta<\frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\theta=\frac{3\sqrt{2}}{5}$. [3]

(b) Hence, find $\sin\theta$. [2]

Point D is loacted on [AC] such that the areau of triangle BCD is $6\sqrt{14}$.

(b) Find DC. [2]

Problem 5 *(JL Nov.2020)* [/5 marks]

Consider the complex numbers $z_1=\cos\frac{11\pi}{12}+\mathrm{i}\sin\frac{11\pi}{12}$ and $z_2=\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}$.

(a) (i) Find $\frac{z_1}{z_2}$

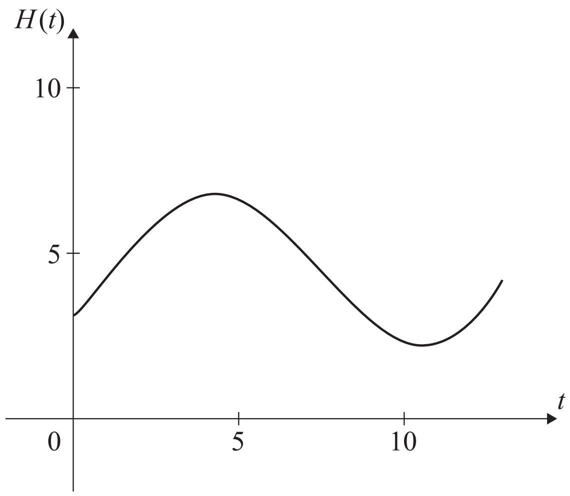
(ii) Find $\frac{z_2}{z_1}$ [3]

(b) 0, $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$ are represented by three points O, A and B respectively on an Argand diagram. Determine the area of the triangle OAB. [2]

Problem 6 *(SL may 2025 !)* [/13 marks]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin\left(b(t-c)\right) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

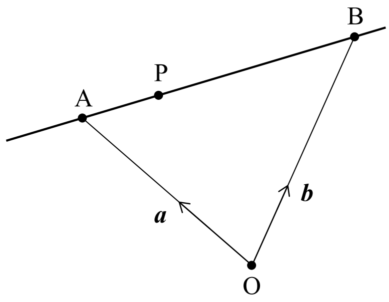
(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

Problem 7 *(HL Nov 2024)* [/8 marks]

The following diagram shows two points A and B such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$.



The point P lies on (AB) so that $\overrightarrow{\mathrm{AP}} = \lambda \overrightarrow{\mathrm{AB}}$ where $0 < \lambda < 1$.

(a) Show that $\overrightarrow{\mathrm{OP}} = (1-\lambda)\boldsymbol{a} + \lambda \boldsymbol{b}$. [1]

It is given that $|\boldsymbol{a}| = 1$, $|\boldsymbol{b}| = 2$ and $\boldsymbol{a} \cdot \boldsymbol{b} = \frac{1}{4}$.

(b) In the case that $\overrightarrow{\mathrm{OP}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$, find the value of λ. [7]