MATHS AA SL

June Exam

PAPER I

Friday 13 June 2025

Duration : 90 min

7 questions

Total :   / 50 marks

Calculator not allowed !

Answers

Problem 1 *(may 2025 !)* **[/7 marks]**

The sum of the first n terms of an *arithmetic* sequence is given by $S_n = pn^2 - qn$,

where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

(a) Find the value of p and the value of q. [5]

$\begin{cases} 40 = 16p - 4q \\ 65 = 25p - 5q \end{cases} \begin{cases} 10 = 4p - q \\ 13 = 5p - q \end{cases} \Rightarrow \boxed{p = 3 \quad \text{and} \quad q = 2}$

(b) Find the value of u_5. $\boxed{u_5 = S_n - S_4 = 25}$ [2]

Problem 2 **[/5 marks]**

The general term of sequence is given by $u_n = \frac{25}{3}(-\frac{2}{5})^n$

i) It is a $\boxed{\textit{geometric}}$ sequence. [1]

ii) The exact expression of u_3 is $\frac{25}{3}(-\frac{8}{125}) = \boxed{-\frac{8}{15}}$ [2]

iii) $3u_n = 4 \quad \Leftrightarrow (-\frac{2}{5})^n = \frac{5}{25} \Leftrightarrow \boxed{n = 2}$ [2]

Problem 3 *(may 2023)* **[/6 marks]**

(a) The equation $\cos(2x) = \sin(x)$ can be written as $1 - 2\sin^2(x) = \sin(x) = 0 \quad \Rightarrow$ ok [1]

(b) $\cos(2x) = \sin(x)$ (where $-\pi \leq x \leq \pi$) $\Leftrightarrow 2s^2 + s - 1 = 0$ [5]

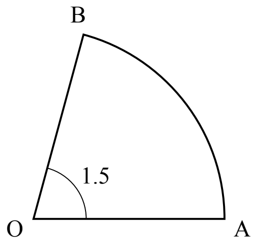
$\Delta = 9 \quad \begin{matrix} s_1 \\ s_2 \end{matrix} = \frac{-1 \pm 3}{4} = \begin{matrix} \frac{1}{2} \Rightarrow \sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \\ -1 \Rightarrow \sin(x) = -1 \Rightarrow x = -\frac{\pi}{2} \end{matrix}$ $\quad \boxed{S = \{-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\}}$

Problem 4 *(may 2025 !)* *[/5 marks]*

Points A and B lie on a circle with centre O and radius r cm, where $\hat{AOB} = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



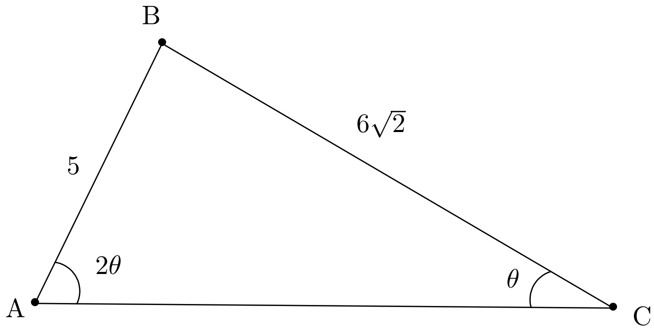
The area of sector OAB is $48\,cm^2$. then $\frac{1}{2}r^2 \times 1.5 = 48$

(a) Find the value of r. $r = \sqrt{64} = \boxed{8cm}$ [3]

(b) Hence, find the perimeter of sector OAB. $p + 2r + r\theta + 16 + 12 = \boxed{28cm}$ [2]

Problem 5 *(may 2025 !)* *[/7 marks]*

The following diagram shows a non-right angled triangle ABC



$AB = 5$, $BC = 6\sqrt{2}$ $\widehat{ACB} = \theta$ and $\widehat{BAB} = 2\theta$ where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule: $\frac{\sin(\theta)}{5} = \frac{\sin(2\theta)}{6\sqrt{2}} \Rightarrow 6\sqrt{2}\sin(\theta) = 10\cos(\theta)\sin(\theta) \Rightarrow \boxed{\cos\,\theta = \frac{3\sqrt{2}}{5}}$

(b) Hence $\sin(\theta) = \sqrt{1 - \left(\frac{3\sqrt{2}}{5}\right)^2} = \sqrt{1 - \frac{18}{25}} = \boxed{\frac{\sqrt{7}}{5}}$

Point D is located on [AC] such that the area of triangle BCD is $6\sqrt{14}$.

(c) Area of triangle BCD= $\frac{1}{2}DC \times BC\,\sin(\theta) \Rightarrow 6\sqrt{14} = \frac{1}{2}DC \times 6\sqrt{2} \times \frac{\sqrt{7}}{5} \Rightarrow \boxed{DC = 10}$

Problem 6 [/7 marks]

(a) The equation $\sin^2 x = \frac{4-5\cos x}{2}$ may be written as $1-\cos^2 x = \frac{4-5\cos x}{2}$ [2]

$\Rightarrow 2-2\cos^2 x = 4-5\cos x \quad \Rightarrow \ 2\cos^2 x - 5\cos x + 2 = 0$.

(b) Hence, we solve $2c^2 - 5c + 2 = 0$ with $c = \cos(x)$ and $0 \leq x \leq 2\pi$. [5]

$\Delta = 9 \quad \begin{matrix} s_1 \\ s_2 \end{matrix} = \frac{5 \pm 3}{4} = \begin{matrix} \frac{1}{2} \Rightarrow \cos(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \\ 2 \Rightarrow \text{impossible} \end{matrix}$ $\boxed{S = \{\frac{\pi}{3}, \frac{5\pi}{3}\}}$

Problem 7 *(may 2025 !)* [/13 marks]

(a) $b = \frac{2\pi}{T} = \boxed{\frac{\pi}{6}}$

(b) $a = \boxed{2.3m}$

(c) $d = \frac{2.2+6.8}{2} = \boxed{4.5m}$

(d) $H(4{:}30) = 6.8 \Rightarrow 2.3\sin\left(\frac{\pi}{6}(4.5-c)\right) + 4.5 = 6.8 \Rightarrow \sin\left(\frac{\pi}{6}(4.5-c)\right) = 1$

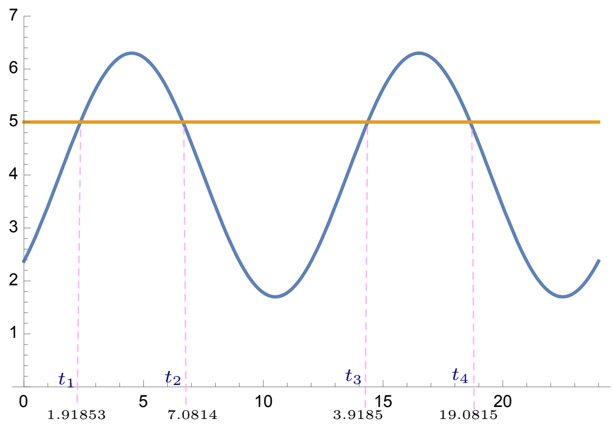
$\Rightarrow \frac{\pi}{6}(4.5-c) = \frac{\pi}{2} \Rightarrow (4.5-c) = 3$

$\Rightarrow \boxed{c = \frac{3}{2}}$

(e) $H(12) = 2.3\sin\left(\frac{\pi}{6}\left(12-\frac{3}{2}\right)\right) + 4.5 = 2.3\sin\left(\frac{\pi}{6}\left(\frac{21}{2}\right)\right) + 4.5 = 2.3\sin\left(\frac{7\pi}{4}\right) + 4.5 = \boxed{2.874m}$

(f) As the period is 12hours, we first find the solutions of $H(t) = h$

with $H(t) = 2.3\sin\left(\frac{\pi}{6}\left(t-\frac{3}{2}\right)\right) + 4.5$ and $h = 5$



We find $t_1 = 2.35905h$, $t_2 = 6.64095h$, $t3 = 14.359h$, $t_4 = 18.641h$

Then $H(t) > h$ for $t \in]t_1, t_2[\cup]t_3, t_4[$ that is during $t_2 - t_1 + t_4 - t_3 = 2 \times 5.16294 = \boxed{10.326h}$