Maths 10

Test 8

subject: *Logarithms*

11 March 2019

Name: ___________

Question 1

Solve :

1) $\log_3(x-4)+\log_3(x-6)=1 \Leftrightarrow (x-4)(x-6)=3^1 \Leftrightarrow x^2-10x+21=0$

$\Delta=16$ the candidates are 3 and 7, but has to be >6 $\boxed{S=\{7\}}$

2) $\log_3(x-4)-\log_3(x-6)=\log_2(22)-\log_2(11) \Leftrightarrow \log_3\left(\frac{x-4}{x-6}\right)=\log_2\left(\frac{22}{11}\right)=1$

Hence $\frac{x-4}{x-6}=3^1=3 \quad \Rightarrow x-4=3x-18 \quad \Rightarrow 2x=14$ $\boxed{S=\{7\}}$

Question 2

Find m such that $\log_2^2(m-1)-5\log_2(m-1)+6=0$

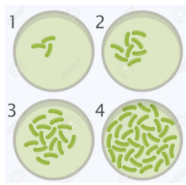
Idea : Taking $x=\log_2(m-1)$ we get the equation $x^2-5x+6=0$

$\Delta=1$ $x=2$ or $x=3$ $\Rightarrow (m-1)=2^2$ or $(m-1)=2^3$ $\Rightarrow S=\{5,9\}$

Question 3

Two populations of bacteria ($\mathcal{B}_1$ and $\mathcal{B}_2$) are growing at different rates.

Their populations at time t are given by:



$n_1(t)=2^{(t+6)}$ for $\mathcal{B}_1$

$n_2(t)=3^{(2t+1)}$ for $\mathcal{B}_2$ (where t is in days).

1) *Initially* (taking $t=0$) $\mathcal{B}_1$ has $2^6=64$ bacteria and $\mathcal{B}_1$ has 3 bacteria only.

2) The value of N is $\boxed{\mathcal{B}_2(3)=3^7=2187}$

3) We want t such that $n_1(t)=N \quad \Rightarrow 2^{(t+6)}= 2187 \quad \Rightarrow$ t+6=$\log_2(2187)=11.0947$

$\Rightarrow \boxed{t=5.0947\text{ days}} = 5\text{days}+0947\times 24h = 5\text{days}+2h+16\text{min}+25.32\text{ sec}$

4) $\mathcal{B}_1$ and $\mathcal{B}_2$ have equal number of bacteria when $n_1(t)=n_2(t)$

$\Rightarrow 2^{(t+6)}=3^{(2t+1)} \Rightarrow t+6=\log_2(3^{(2t+1)})=(2t+1)\log_2(3) \Rightarrow t(1-2\log_2(3))=\log_2(3)-6$

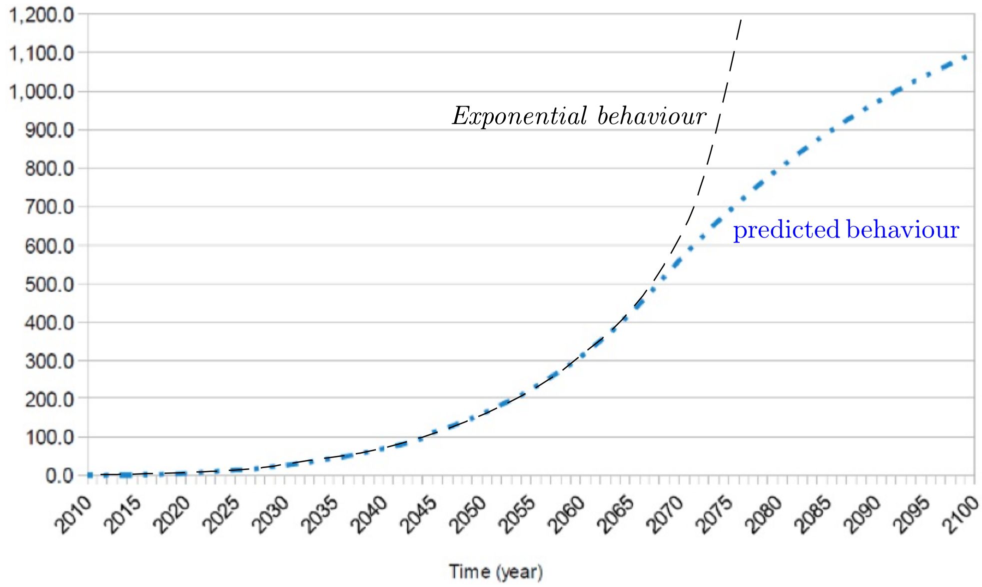
$\Rightarrow t=\frac{6-\log_2(3)}{2\log_2(3)-1}=\boxed{2.035\text{ day}}$ (about 2 days and 50mim)

Question 4

The figure below shows the sea leve rise prevision according to a study published in November 2012 by two Climate scientist James Hansen and Makiko Sato.

Fig.1

Elevation $\boldsymbol{h}$ of sea level (in cm) above the level of 2009.



Let be $\boldsymbol{n}$ the number of year after 2000 (for exemple for the year 2019, n is 19)

and assume that the elevation is given by : $h(n) = 3.6 \times 1.077^n$.

1) Complete the following table :

year	$\boldsymbol{n}$	see level $\boldsymbol{h}$ (cm)
2045	45	$3.6 \times 1.077^{45} \cong$ 101.4
2055	55	$3.6 \times 1.077^{55} \cong 213$
2060	60	$3.6 \times 1.077^{60} \cong 308$

2) Using the formula, find in which year h will be 400 cm ?

Solving $h(n) = 3.6 \times 1.077^n = 400 \quad \Rightarrow n = \log_{1.077}\left(\frac{400}{3.6}\right) \cong 63.5$ then: $\boxed{\text{middle of year 2063}}$

Bonus

According to the formula, in 2090 the elevation $\boldsymbol{h}$ would be $3.6 \times 1.077^{90} = 2855.7$cm.

The value provided by the curve above is about 960cm, that its much less.

The reason why the values are so different is that after 2070, the curve stop growing like an *exponential*, hence the formula doesn't match anymore the curve.

(The figure below compares the pure *exponential behaviour* of the formula with behaviour predicted in the article of Hansen & Sato).

The reason why the elevation will stop increasing exponentially after some years is maybe that it will have almost no more melting ice in the glaciers !