Maths 10

Test 7

subject: *system of simultaneous equations*

31 January 2019

ANSWERS

Problem 1

Solve the following systems of simultaneous equations :

1) by *substitution* (x from I) : [5 marks]

$$\begin{cases} x+3y=32 \quad \Rightarrow x=32-3y=\ldots=32-3\times 4=20 & \text{(I)} \\ 5x-7y=72 \Rightarrow 5(32-3y)-7y=72 \Rightarrow y=\frac{160-72}{22}=4 & \text{(II)} \end{cases}$$

$S=\{(20,4)\}$

2) by *combination** [5 marks]

$$\begin{cases} \frac{x}{2}+5y=-5 & \text{(I)} \\ x-\frac{3y}{2}=-13 & \text{(II)} \end{cases} \Rightarrow \begin{cases} x+10y=-10 & \text{(2 I)} \\ x-\frac{3y}{2}=-13 & \text{(II)} \end{cases}$$

$$2\ \text{I} -\ \text{II} : \left(10+\frac{3}{2}\right)y=3 \quad \Rightarrow y=3\times\frac{2}{23}=\frac{6}{23}$$

$S=\left\{\left(\frac{-290}{23},\frac{32}{23}\right)\right\}$

3) by *Cramer* [5 marks]

$$\begin{cases} \sqrt{27}x+\sqrt{2}y=7 & \text{(I)} \\ \sqrt{8}x+\sqrt{3}y=\sqrt{6} & \text{(II)} \end{cases} \qquad D=\begin{vmatrix} \sqrt{27} & \sqrt{2} \\ \sqrt{8} & \sqrt{3} \end{vmatrix}=5$$

$$D_x=\begin{vmatrix} 7 & \sqrt{2} \\ \sqrt{6} & \sqrt{3} \end{vmatrix}=5\sqrt{3} \quad D_y=\begin{vmatrix} \sqrt{27} & 7 \\ \sqrt{8} & \sqrt{6} \end{vmatrix}=-5\sqrt{2}$$

$S=\{\sqrt{3},-\sqrt{2})\}$

4) by the method you want [5 marks]

$$\begin{cases} \pi x-3y=-8 & \text{(I)} \\ -2x+y=2 & \text{(II)} \end{cases} \qquad \text{I} +3\ \text{II} : x=\frac{-2}{\pi-6} \quad \text{and } y=2+2x$$

$S=\left\{\left(\frac{2}{6-\pi},\frac{16-2\pi}{6-\pi}\right)\right\}$

Problem 2

i) What is a *singular* system ? It is a system having either *no solution*, or *infinite solutions*. Such a system has D=0

ii) Which of the three following systems are *singular*? [3 marks]

A) $\begin{cases} 18x-12y=78 \\ -15x+10y=-65 \end{cases}$ B) $\begin{cases} -3x-21y=39 \\ 2x-14y=26 \end{cases}$ C) $\begin{cases} 3x-21y=19 \\ -2x+14y=-4 \end{cases}$

ii) State and explain, about the singular system of (i) if it has an *infinite number of solutions* or *no solutions*.

A) : both equations equivalent to $3x-2y=13 \Rightarrow$ infinite solutions

C) :incompatible equations $\Rightarrow$ No solutions

Problem 3 [8 marks]

Solve the following system of *simultaneous equations*, giving x and y in terms of k

$$\begin{cases} x-ky=1 \\ -5x+6y=-2 \end{cases} \quad D=\begin{vmatrix} 1 & -k \\ -5 & 6 \end{vmatrix}=6-5k \quad D_x=\begin{vmatrix} 1 & -k \\ -2 & 6 \end{vmatrix}=6-2k \quad D_y=\begin{vmatrix} 1 & 1 \\ -5 & -2 \end{vmatrix}=3$$

Conclusion : $\boxed{x=\frac{D_x}{D}=\frac{6-2k}{6-5k}}$ and $\boxed{y=\frac{D_y}{D}=\frac{3}{6-5k}}$ $S=\left\{\left(\frac{2-2k}{2-5k},\frac{3}{2-5k}\right)\right\}$

<u>Bonus</u>

i) – Find the value of k (of problem 3) for having the solution $x=3$. [+2 marks]

$$x=3 \Rightarrow \frac{6-2k}{6-5k}=3 \Rightarrow 6-2k=18-15k \Rightarrow 13k=12 \quad \Rightarrow \boxed{k=\frac{12}{13}}$$

– Hence the value for y would be: $y=\frac{3}{6-5\frac{12}{13}}=\frac{3\times13}{78-60}=\boxed{\frac{39}{18}}$

ii) – Find the value of k such that the system (as given in problem 3) is *singular*. [+2 marks]

$$D=0 \Rightarrow 6-5k=0 \Rightarrow \boxed{k=\frac{6}{5}}$$

– How many solution(s) would have this singular system ?

with $k=\frac{6}{5}$ the sytem is : $\begin{cases} x-\frac{6}{5}y=1 \\ -5x+6y=-6 \end{cases} \Rightarrow \begin{cases} 5x-6y=5 \\ 5x-6y=-6 \end{cases}$ $5\neq-6$: no solutions !