Test 6

Friday Jan.25th 2019

Maths 10

Revision Second degree & Parametric equations ANSWERS

1) Solving second degree equation

$$6x^2-x=2 \Leftrightarrow 6x^2-x-2=0 \Rightarrow \left|\begin{array}{l} a=6 \\ b=-1 \\ c=-2 \end{array}\right. \quad \Delta=b^2-4\text{ac}=49 \quad \boxed{\text{S}=\{-\tfrac{1}{2},\tfrac{2}{3}\}}$$

2) Second degree parametric equations

i) The equation $3x^2-mx+12=0$ has *only one* solution if $m^2-144=0 \Rightarrow \boxed{m\in\{-12,12\}}$

ii) The equation $px^2-6x+p=0$ has *only one* solution if $36-4p^2=0 \Rightarrow \boxed{p\in\{-3,3\}}$

iii) The equation $9x^2-6sx+4s=0$ has *two* solutions if $36s^2-144s>0 \Leftrightarrow 46s(s-4)>0$

s	-∞		0		4		∞
46s		$-$	0	$+$		$+$	
$(1-4s)$		$-$		$-$	0	$+$	
$46s(s-4)$		$+$		$-$		$+$	

$\boxed{s\in\ ]-\infty,0[\ \cup\ ]4,\infty[}$

$$S=\left\{\frac{6s-\sqrt{36-144s^2}}{18},\frac{6s-\sqrt{36+144s^2}}{18}\right\}$$
$$=\left\{\frac{s-\sqrt{1-4s^2}}{3},\frac{s+\sqrt{1-4s^2}}{3}\right\}$$

3) Factorisation $\boxed{ax^2+bx+c=a(x-x_1)(x-x_2)}$

iv) $6x^2-x-2=6\left(x+\frac{1}{3}\right)\left(x-\frac{2}{3}\right)=(3x+2)(3x-2)$

iv) $9x^2-6sx+4s=9\left(x-\frac{1-\sqrt{1-4s^2}}{3}\right)\left(x-\frac{1+\sqrt{1-4s^2}}{3}\right)$

$$=\left(3x-1-\sqrt{1-4s^2}\right)\left(x-1+\sqrt{1-4s^2}\right)$$

4) The equation $mx^2-8x+m=0$ has *two* solution if $64-4m^2>0 \Leftrightarrow m^2<4^2$

By the rule If $m^2<A^2$ then $-A<m<A$) we find : $\boxed{m\in\ ]-4,4[}$

5) i) The equation $(\lambda-1)x^2+(\lambda+1)x+(\lambda-1)=0$ have *an unique* (a double) solution if $\Delta=(\lambda+1)^2-4(\lambda-1)^2=0 \Rightarrow \lambda^2+2\lambda+1-4(\lambda^2-2\lambda+1)=0 \quad \Rightarrow -3\lambda^2+10\lambda-3=0$

we solve ths last equation by using an other delta ! $\Delta_\lambda=10^2-4(-3)(-3)=64$

then $\boxed{\lambda\in\{\frac{1}{3},3\}}$ Notice : For these values of λ, $\Delta=0$, and $x=-\frac{b}{2a}=-\frac{\lambda+1}{2(\lambda-1)}$

ii) For $\lambda=\frac{1}{3}$, the *unique* solution is $x=-\frac{\frac{1}{3}+1}{2\left(\frac{1}{3}-1\right)}=-\frac{\frac{4}{3}}{2\left(-\frac{2}{3}\right)}=\boxed{1}$

For $\lambda=3$, the *unique* solution is $x=-\frac{3+1}{2(3-1)}=-\frac{4}{2(2)}=\boxed{-1}$

Bonus The *sum* of the two solutions of 2(iii) is $\frac{s-\sqrt{1-4s^2}}{3}+\frac{s+\sqrt{1-4s^2}}{3}=\boxed{\frac{2s}{3}}$

The formula $s=-\frac{b}{a}$ gives the same result : $-\frac{b}{a}=-\frac{-6s}{9}=\boxed{\frac{2s}{3}}$