Maths 10

Test 5

Tuesday 4^{th} Dec 2018

Inequations

ANSWERS

Problem 1

i) $(k-3)x^2+4(k-9)x+(k-3)$

has a *double solution* $\Leftrightarrow \Delta=0$

$$\Leftrightarrow 16(k-9)^2-4(k-3)^2=0$$

$$\Leftrightarrow 4(k-9)^2-(k-3)^2=0$$

$$\Leftrightarrow 4k^2-72k+324-k^2+6k-9=0$$

$$\Leftrightarrow 3k^2-66k+315=0$$

$$\Leftrightarrow k^2-22k+105=0$$

$\Delta_\lambda=(-22)^2-4(1)(105)$

$=484-420=64 \qquad \Rightarrow \begin{cases} k_1=\frac{22-8}{2}=7 \\ k_1=\frac{22+8}{2}=15 \end{cases}$

$$\boxed{S=\{7,15\}}$$

ii) For each of these values of λ, what is the solution ?

For $k=k_1=7$,the equation $(k-3)x^2+4(k-9)x+(k-3)=0$ becomes
$4x^2-8x+4=0$
$x^2-2x+1=0$
$(x-1)^2=0$

$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=1$

For $k=k_2=15$, the equation $(k-3)x^2+4(k-9)x+(k-3)=0$ becomes
$12x^2+24x+12=0$
$x^2+2x+1=0$
$(x+1)^2=0$

$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=-1$

Problem 2 (à vérifier)

i) $3qx^2-(q+3)x+1=0$ has a *double solution* $\Leftrightarrow \Delta=0$

$$\Leftrightarrow (q+3)^2-4(3q)(1)=0$$

$$\Leftrightarrow q^2+6q+9-12q=0$$

$$\Leftrightarrow q^2-6q+9=0$$

$$\Leftrightarrow (q-3)^2=0$$

$$\boxed{S=\{3\}}$$

ii) *No solution* $\Leftrightarrow (q-3)^2<0$

but for any k, $(q-3)^2\geqslant 0$,

$\boxed{S=\varnothing}$

iii) *Two solutions* $\Leftrightarrow (q-3)^2<0$

but $(q-3)^2>0$ for any k exept 3

$\boxed{S=\mathbb{R}\backslash\{3\}}$

Problem 3

$\lambda x^2 + 8x + 4\lambda = 0$ has two solutions $\Leftrightarrow \Delta > 0$

$\Leftrightarrow (8)^2 - 4(\lambda)(4\lambda) > 0$

$\Leftrightarrow 64 - 16\lambda^2 > 0 \quad \Leftrightarrow \quad 64 > 16\lambda^2 \Leftrightarrow \lambda^2 < 4$

According to the rule : $m^2 < A^2 \quad \Rightarrow -A < m < A$, (here with $A^2 = 4$)

$\boxed{S =]-2, 2[}$