Maths 10

Pre - Test 8

subject: *Logarithms*

5 March 2019

Answers

Question 1

$\log_7(80x^2 - 144x + 55) = 3 \quad \Rightarrow (80x^2 - 144x + 55) = 7^3 \ \Rightarrow 5x^2 - 9x - 18 = 0$

$\Delta = 441 \quad x = \frac{9 \pm 21}{10} \ = 3$ or $-\frac{6}{5}$ two candidates, only the first is accepted $\Rightarrow \boxed{S = \{3\}}$

Question 2

Solve $\log_2(3x+17) - \log_2(8x-36) = \log_2(24) - \log_2(3)$

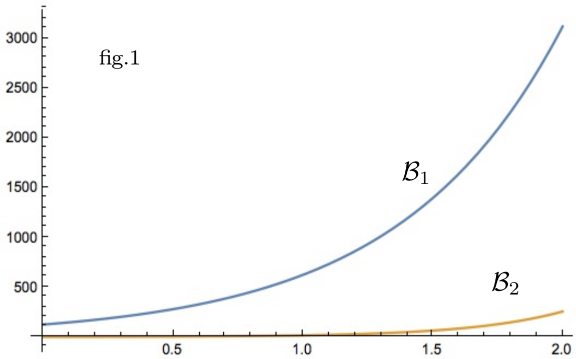
$\log_7\left(\frac{3x+17}{8x-36}\right) = \log_7\left(\frac{24}{3}\right) \quad \Rightarrow \frac{3x+17}{8x-36} = 8 \quad \Rightarrow 3x+17 = 8(8x-36) \ \Rightarrow x = \frac{305}{61} = 5 \quad \boxed{S = \{5\}}$

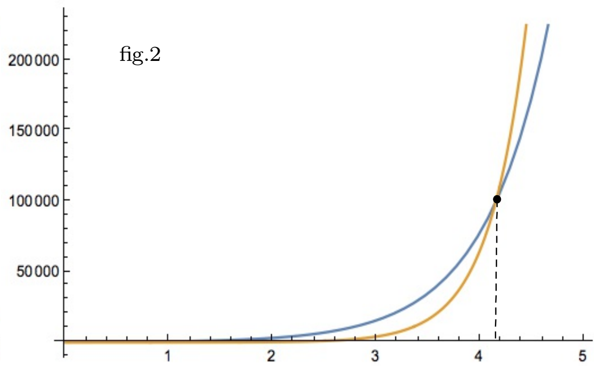
Question 3

$\mathcal{B}_1 : n_1(t) = 5^{(t+3)} \qquad \mathcal{B}_2 : n_2(t) = 4^{(2t)}$

i) comparing at $t = 1$ we have : $\mathcal{B}_1$ with $n_1(1) = 5^4 = 625$ bacteria and $\mathcal{B}_2$ with $n_2(1) = 4^2 = 16$ only.

But at $t = 5$ we have : $\mathcal{B}_1$ with $n_1(5) = 5^8 = 39\ 065$ and $\mathcal{B}_2$ with $n_2(1) = 4^{10} = 1\ 048\ 576$ bacteria.





ii) The populations the same at time t such that $n_1(t) = n_2(t)$

Hence we solve $5^{(t+3)} = 4^{(2t)} \quad \Rightarrow \log_5(5^{(t+3)}) = \log_5(4^{(2t)})$

$\Rightarrow t + 3 = 2t\log_5(4) \Rightarrow t = \frac{3}{2\log_5(4) - 1} = \frac{3}{2\frac{\log(4)}{\log(5)} - 1} = \boxed{4.151 \text{ days}}$

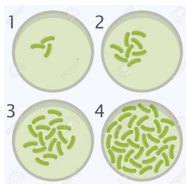


fig.3

Question 4

$N_o =$ 17 grams (initial mass)

$N_n = N_o\left(\frac{1}{2}\right)^n$ where $n = 0, 1, 2, 3$ is the number $\boxed{138 \text{ days}}$.

i) $N_o\left(\frac{1}{2}\right)^n = \frac{1}{8} = \left(\frac{1}{2}\right)^3$ for $n = 3$, hence after $3 \times 138 = \boxed{414 \text{ days}}$.

ii) $N_o\left(\frac{1}{2}\right)^n = 1\text{g} \Rightarrow \left(\frac{1}{2}\right)^n = \frac{1}{N_o} = \frac{1}{12} \Rightarrow n = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{12}\right) = \frac{\log\left(\frac{1}{12}\right)}{g\left(\frac{1}{2}\right)} = 3.585 \Rightarrow$ about $\boxed{495 \text{ days}}$.

iii) $N_o\left(\frac{1}{2}\right)^n = (100 - 99)\%$ of $N_o \quad \Rightarrow \left(\frac{1}{2}\right)^n = \frac{1}{100} \Rightarrow n = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{100}\right) = \frac{\log\left(\frac{1}{100}\right)}{g\left(\frac{1}{2}\right)} = 6\text{-}644 \Rightarrow \boxed{495 \text{ days}}$.