Maths IB_1

Pre - Test 7

subject: *System of simultaneous equations*

25 January 2019

ANSWERS

Problem 1

Solve the following systems of simultaneous equations :

1) by *substitution* (x from I)

$$\begin{cases} x+3y=294 \Rightarrow x=294-3y=\ldots=294-3\times 100=-6 & \text{I} \\ 25x+y=-50 \Rightarrow 25(294-3y)+y=-50 \Rightarrow y=\frac{7350+50}{37}=100 & \text{II} \end{cases}$$

$S=\{(-6,100)\}$

2) by *combination**

$$\begin{cases} \frac{x}{2}+6y=17 & \text{I} \\ -x-\frac{3y}{2}=\frac{11}{4} & \text{II} \end{cases} \Rightarrow \begin{matrix} 4\text{ I} \\ 4\text{ II} \end{matrix} \begin{cases} 2x+24y=68 \\ -4x-6y=11 \end{cases} \Rightarrow \begin{matrix} 8\text{ I} \\ 4\text{ II} \end{matrix} \begin{cases} 4x+48y=136 \\ -4x-6y=11 \end{cases}$$

$S=\left\{\left(-8,\frac{7}{2}\right)\right\}$

$$8\text{ I} + 4\text{ II} : 42y=147 \Rightarrow y=\frac{147}{42}=\frac{7}{2}$$

3) by *Cramer*

$$\begin{cases} \sqrt{27}x+\sqrt{2}y=-\sqrt{2} & \text{I} \\ \sqrt{8}x+\sqrt{3}y=\sqrt{12} & \text{II} \end{cases} \qquad D=\begin{vmatrix} \sqrt{27} & \sqrt{2} \\ \sqrt{8} & \sqrt{3} \end{vmatrix}=5$$

$$D_x=\begin{vmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{12} & \sqrt{3} \end{vmatrix}=-3\sqrt{6} \quad D_y=\begin{vmatrix} \sqrt{27} & -\sqrt{2} \\ \sqrt{8} & \sqrt{12} \end{vmatrix}=18+4=22$$

$S=\left\{\left(-\frac{3\sqrt{6}}{5},\frac{22}{5}\right)\right\}$

4) by the method you want

$$\begin{cases} 3x+5y+1=0 & \text{I} \\ 5x+7y+11=0 & \text{II} \end{cases} \Rightarrow \begin{matrix} 5\times\text{I} \\ 3\times\text{II} \end{matrix} \begin{cases} 15x+25y+5=0 \\ 15x+21y+33=0 \end{cases} \Rightarrow 5\times\text{I}-3\times\text{II}: \ 4y-28=0 \Rightarrow y=7$$

$S=\{(-12,7)\}$

5) by the method you want

$$\begin{cases} \pi x-3y=-8 & \text{I} \\ -2x+y=2 & \text{II} \end{cases} \qquad \text{I} +3\text{ II} : x=\frac{-2}{\pi-6} \text{ and } y=2+2x$$

$S=\left\{\left(\frac{-2}{\pi-6},\frac{-16+2\pi}{\pi-6}\right)\right\}$

⋆ An other covenient combination forn(2) is I $+\frac{1}{2}$II

Problem 2

i) What is a *singular* system ? It is a system having either *no solution*, or *infinite solutions*. Such a system has D=0

ii) Which of the three following systems is *singular* ? [3 marks]

A) $\begin{cases} x - \frac{1}{3}y = -8 \\ -3x + 2y = 2 \end{cases}$ B) $\begin{cases} \frac{\pi}{2}x - 3y = -8 \\ 0x + y = 2 \end{cases}$ C) $\begin{cases} 3x - 21y = 19 \\ -2x + 14y = -4 \end{cases}$ C

ii) Which of the three following *singular* systems has an *infinite number of solutions* ? [3 marks]

A) $\begin{cases} 12x - 8y = 52 \\ 15x - 10y = 65 \end{cases}$ B) $\begin{cases} 3x - 21y = 26 \\ -2x + 14y = 52 \end{cases}$ C) $\begin{cases} -3x - 21y = -52 \\ 2x + 14y = 26 \end{cases}$ A

Problem 3

[8 marks]

Solve the following system of simultaneous equations, giving x and y in terms of k

$$\begin{cases} x - ky = 1 \\ -3x + 2y = -6 \end{cases} \quad D = \begin{vmatrix} 1 & -k \\ -3 & 2 \end{vmatrix} = 2 - 3k \quad D_x = \begin{vmatrix} 1 & -k \\ -6 & 2 \end{vmatrix} = 2 - 6k \quad D_y = \begin{vmatrix} 1 & 1 \\ -3 & -6 \end{vmatrix} = -3$$

Conclusion : $\boxed{x = \frac{D_x}{D} = \frac{2-6k}{2-3k}}$ and $\boxed{y = \frac{D_y}{D} = \frac{-3}{2-3k}}$ $S = \left\{\left(\frac{2-6k}{2-3k}, \frac{-3}{2-3k}\right)\right\}$

Bonus

i) – Find the value of k (of problem 3) for having the solution $x = 3$. [+2 marks]

$x = 3 \Rightarrow \frac{2-6k}{2-3k} = 3 \Rightarrow 2 - 6k = 6 - 9k \Rightarrow 3k = 4 \Rightarrow \boxed{k = \frac{4}{3}}$

– Hence the value for y would be $y = \frac{-3}{2 - 3\frac{4}{3}} = \frac{-3}{-2} = \boxed{\frac{3}{2}}$

ii) – Find the value of k such that the system (as given in problem 3) is *singular*. [+2 marks]

$D = 0 \Rightarrow 2 - 3k = 0 \Rightarrow \boxed{k = \frac{2}{3}}$

– How many solution(s) would have this singular system ?

with $k = \frac{2}{3}$ the sytem is : $\begin{cases} x - \frac{2}{3}y = 1 \\ -3x + 2y = -6 \end{cases} \Rightarrow \begin{cases} 3x - 2y = 3 \\ -3x + 2y = -6 \Rightarrow 3x - 2y = 6 \end{cases}$ $3 \neq 6$: $\boxed{\text{no solutions !}}$