Pre-Test Revision

Maths 10 — Monday 21th Jan 2019

second degree and parametric equation — ANSWERS

Problem 1 (Solving second degree equations is finding the set S of the solutions)

i) $4x^2-7x-15=0 \qquad \Delta=49+240=289 \qquad S=\left\{\frac{7\pm 17}{8}\right\}=\left\{\frac{-5}{4},3\right\}$

ii) $2x^2-8x+8=0 \qquad \Delta=64-64=0 \qquad S=\left\{\frac{-b\pm 0}{2a}\right\}=\{2\}$

iii) $2x^2-5x+\frac{13}{2}=0 \qquad \Delta=25-26<0 \qquad S=\varnothing$

Problem 2 The number of solutions for each of the following equations is ...

i) $3x^2-30x+78=6(x-5) \Rightarrow 3x^2-36x+108=0 \Rightarrow x^2-12x+36=0\ \Delta=144-144=0 \Rightarrow$ one

ii) $2x^2-7(2x-4)=0 \Rightarrow 2x^2-14x+28=0 \Rightarrow x^2-7x+14=0\ \Delta=49-56<0 \Rightarrow$ zero

iii) $2x^2-7(2x+4)=0 \Rightarrow 2x^2-14x-28=0 \Rightarrow x^2-7x-14=0\ \Delta=49+56>0 \Rightarrow$ two

Problem 3

i) $x^2-15x+44=(x-11)(x-4)$

iv) $2x^2-5x+3=(x-1)(2x-3)$

Problem 4

$mx^2-3x+m=0$ has two solutions $\Leftrightarrow \Delta>0$

$\Leftrightarrow (-3)^2-4(m)(m)>0$

$\Leftrightarrow 9-4m^2>0 \quad \Leftrightarrow -4m^2>-9 \quad \Leftrightarrow 4m^2<9 \quad \Leftrightarrow m^2<\frac{9}{4}$

According to the rule : $m^2<A^2 \Rightarrow -A<m<A$, (here with $A^2=\frac{9}{4}$) $\qquad S=\left]-\frac{3}{2},\frac{3}{2}\right[$

Problem 5

i) $(s-1)x^2-2(s-3)x+s-3=0$ has a *double solution* $\Leftrightarrow \Delta=0$

$\Leftrightarrow 4(s-3)^2-4(s-1)(s-3)=0$

$\Leftrightarrow 4s^2-24s+36-4s^2+16s-12=0$

$\Leftrightarrow -8s+24=0 \Leftrightarrow 8s=24 \Leftrightarrow s=3$

$S=\{3\}$

ii) $(s-1)x^2-2(s-3)x+s-3=0$ has a *no solution* $\Leftrightarrow \Delta<0$

$\Leftrightarrow -8s+24<0 \quad \Leftrightarrow 8s>24 \quad \Leftrightarrow s>3$

$S=]3,\infty[$

Problem 6

i) $(\lambda+2)x^2+4(\lambda-4)x+(\lambda+2)=0$ has a *double solution* $\Leftrightarrow \Delta=0$

$$\Leftrightarrow 16(\lambda-4)^2-4(\lambda+2)^2=0$$
$$\Leftrightarrow 16\lambda^2-128\lambda+256-4\lambda^2-16\lambda-16=0$$
$$\Leftrightarrow 12\lambda^2-144\lambda+240=0$$
$$\Leftrightarrow \lambda^2-12\lambda+20=0$$

$$\Delta_\lambda=(-12)^2-4(1)(20)$$
$$=144-80=64 \qquad \Rightarrow \begin{cases} \lambda_1=\frac{12-8}{2}=2 \\ \lambda_1=\frac{12+8}{2}=10 \end{cases}$$

$$\boxed{S=\{2,10\}}$$

ii) For each of these values of λ, what is the solution ?

For $\lambda=\lambda_1=2$, the equation $(\lambda+2)x^2+4(\lambda-4)x+(\lambda+2)=0$ becomes $4x^2-8x+4=0$
$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=1$ $x^2-2x+1=0$
$(x-1)^2=0$

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For $\lambda=\lambda_2=10$, the equation $(\lambda+2)x^2+4(\lambda$ becomes $12x^2+24x+12=0$
$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=-1$ $x^2+2x+1=0$
$(x+1)^2=0$