Maths 10

Pre-Test 5

Friday Nov.30th 2018

Inequations

ANSWERS

Problem 1

$mx^2-3x+m=0$ has two solutions $\Leftrightarrow \Delta>0$

$\Leftrightarrow (-3)^2-4(m)(m)>0$

$\Leftrightarrow 9-4m^2>0 \quad \Leftrightarrow -4m^2>-9 \quad \Leftrightarrow 4m^2<9 \quad \Leftrightarrow m^2<\frac{9}{4}$

According to the rule : $m^2<A^2 \quad \Rightarrow -A<m<A$, (here with $A^2=\frac{9}{4}$) $\quad \boxed{S=]-\frac{3}{2},\frac{3}{2}[}$

Problem 3

i) $(s-1)x^2-2(s-3)x+s-3=0$ has a *double solution* $\Leftrightarrow \Delta=0$

$\Leftrightarrow 4(s-3)^2-4(s-1)(s-3)=0$

$\Leftrightarrow 4s^2-24s+36-4s^2+16s-12=0$

$\Leftrightarrow -8s+24=0 \Leftrightarrow 8s=24 \Leftrightarrow s=3$

$\boxed{S=\{3\}}$

ii) $(s-1)x^2-2(s-3)x+s-3=0$ has a *no solution* $\Leftrightarrow \Delta<0$

$\Leftrightarrow -8s+24<0 \quad \Leftrightarrow 8s>24 \quad \Leftrightarrow s>3$

$\boxed{S=]3,\infty[}$

Problem 4

i) $(\lambda+2)x^2+4(\lambda-4)x+(\lambda+2)=0$ has a *double solution* $\Leftrightarrow \Delta=0$

$\Leftrightarrow 16(\lambda-4)^2-4(\lambda+2)^2=0$

$\Leftrightarrow 16\lambda^2-128\lambda+256-4\lambda^2-16\lambda-16=0$

$\Leftrightarrow 12\lambda^2-144\lambda+240=0$

$\Leftrightarrow \lambda^2-12\lambda+20=0$

$\Delta_\lambda=(-12)^2-4(1)(20)$

$=144-80=64 \qquad \Rightarrow \begin{cases} \lambda_1=\frac{12-8}{2}=2 \\ \lambda_1=\frac{12+8}{2}=10 \end{cases}$

$\boxed{S=\{2,10\}}$

ii) For each of these values of λ, what is the solution ?

For $\lambda=\lambda_1=2$, the equation $(\lambda+2)x^2+4(\lambda-4)x+(\lambda+2)=0$ becomes $4x^2-8x+4=0$, $x^2-2x+1=0$, $(x-1)^2=0$

$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=1$

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For $\lambda=\lambda_2=10$, the equation $(\lambda+2)x^2+4(\lambda$ becomes $12x^2+24x+12=0$, $x^2+2x+1=0$, $(x+1)^2=0$

$\Delta=0$ (as expected) and $x_1=x_2=-\frac{b}{2a}=-1$